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Cluster Detection in Laboratory Auction Data: A Model-Based Approach

Summary: The benchmark risk-averse equilibrium model does not explain some of the outcomes obtained in experiments with first-price auctions. Nonetheless, the presence of non-linear bidding and the wide dispersion of bids have received little attention in the literature. I focus on these issues and revisit previous laboratory evidence with the help of model-based clustering techniques. The rejection of equilibrium models is found to be mostly due to the significance of non-linear bidding rules and the unexplained heterogeneity. With the use of a mixture model, the observations are classified into four groups or clusters. Significant differences between individuals and clusters are found, but so is a persistent *within individual* variation, which leads us to conclude that subjects do not commit to one particular bidding strategy and alternate across several processes.

Key words: Clustering, Experimental Auctions, Non-linear bidding.

JEL: C91, D44, D03.

In the empirical auctions literature a well-known result is that the subjects in experiments of independent first-price private-value auctions do not bid as predicted by risk-neutral equilibrium (RNE). As stated in Richard Engelbrecht-Wiggans and Elena Katok (2007, p. 82) "this result has been replicated numerous times by different researchers at different laboratories and under a variety of environments". The reason for this is the prevalence of some phenomena in the data that cannot be explained by the benchmark Nash equilibrium models of bidding. First, individuals tend to make bids above the predicted RNE bid – henceforth, *overbidding*. Second, uniformly valued bidders make bids that are not proportional to their valuations – henceforth, *non-linearity*. Third, bids display high volatility around the mean bidding function – henceforth, *overdispersion*.

From these, researchers have concentrated their efforts on solving the puzzle posed by overbidding. Researchers first explored the role of risk aversion in the equilibrium bid function.¹ Thus, Cox, Bruce Roberson, and Smith (1982) propose a mod-

¹ An extensive literature explores several alternative hypotheses besides risk aversion, such as individual interaction (Mark Isaac and James Walker 1985; John Morgan, Ken Stiglitz, and George Reis 2003), learning direction theory (Wiggans 1989; Axel Ockenfels and Reinhard Selten 2005; Tibor Neugebauer and Selten 2006), step-level reasoning (Uri Gneezy 2005), joy of winning (James Cox, Vernon Smith, and James Walker 1983), biased incentives (Glenn Harrison 1989), and Quantal Response Equilibrium (Jacob Goeree, Charles Holt, and Thomas Palfrey 2002). Most of these models assume bounded-rationality on the behalf of agents, and some of them provide testable restrictions aiming to explain the phenomenon of overbidding.

el in which individuals display utility functions with constant relative risk aversion (CRRAM) and find that the model fits the experimental data reasonably well. Later on, Kay-Yut Chen and Charles R. Plott (1998) also find that CRRAM performs better than many other simpler non-fully rational models. However, they also propose a *sophisticated ad-hoc* model with an even better fit. This model specifies a non-linear bidding function for the agents, thus challenging the linear bidding specification.

More recently, the non-linearity of the bidding function has been further explored in Olivier Armantier and Nicolas Treich (2006). These authors postulate a downward bias in how the individuals perceive their winning prospects in an auction (*beliefs*). The authors conduct an auction experiment in which the treatment of beliefs and preferences is separated and find that according to their proposition, subjects consistently underestimate their chances of winning, which explains why the bidding function is not linear in valuations. In an independent work, Paul Pezanis-Christou and Andrés Romeu (2002) propose to estimate the equilibrium bidding model using a Beta distribution for the valuations instead of the standard uniform. When valuations are Beta distributed, the corresponding equilibrium bidding function is non-linear and concave. The authors conclude that this could be the consequence of a misperception of the subject's chances of winning. Finally, Patrick Bajari and Ali Hortacsu (2005) examine the data in Douglas Dyer, John Kagel, and Dan Levin (1989) to assess the performance of different structural equilibrium models. They find that the risk aversion model outperforms not only the risk neutral but also equilibrium models with learning. Besides that, the Quantal Response Equilibrium model, in which individuals are allowed to make errors in equilibrium, provides a remarkably good fit under a Beta distribution.

Apart from the observed non-linearity, it is often the case that conditional on valuations, the bids do not lie close to the expected bidding function. The standard way to deal with this problem is the addition of an error term that incorporates any potential source of heterogeneity, such as sampling error, individual and/or session differences, computing errors, or the learning process. In some cases, however, the unexplained variability is too big to be ignored and we should try to find at least a clue on the nature of this *overdispersion*.

In this paper, we aim to provide a deeper insight on the explanatory power of the equilibrium models regarding the two phenomena of non-linearity and overdispersion, and why they may fail. The data come from two different experimental auction data sources: Dyer, Kagel, and Levin (1989), and Cox, Smith, and Walker (1988). From a methodological point of view, the paper is novel in that we use a model-based clustering technique to analyze the data that to our knowledge had never been used before in the context of empirical auctions.

We find that the risk aversion equilibrium model is rejected on the basis of a significant non-linear bidding and the unexplained *overdispersion* mostly coming from individual *between* and *within* heterogeneity. Put simply, different bidders use different bidding strategies, but also some bidders change their strategy several times during one session. This justifies the use of a clustering technique that lets the data determine the optimal characterization of groups or clusters of observations. Clustering is then used to identify groups of homogeneous bidding and to discriminate between recognizable patterns of bidding strategies.

A model-based clustering approach and the expectation-maximization (EM) algorithm are used to find a representation of the data in terms of a mixture of densities. Groups of observations or clusters in the data are identified using the posterior probability of sampling from each group. Four clusters are found to provide the best representation of the data in terms of fit. Some clusters tend to capture noisy bidding, but most of the observations are classified into clusters characterized by concave non-linear bidding above the RNE prediction. Finally, we find significant differences of posterior cluster probability between individuals, but they do not perfectly classify observations.

The rest of the paper is organized as follows. In Section 1 we briefly explain the equilibrium models of bidding. In Section 2 we present the data and perform an initial empirical analysis. Section 3 contains a brief description of the mixture model and the results of estimation. Section 4 presents some final conclusions.

1. Equilibrium Bidding with Risk Aversion

In a first-price sealed-bid auction, bidders compete for the purchase of a single commodity. The good is awarded to the highest bidder for a price equal to her bid. Each bidder $i=1,\dots,N$ is assumed to receive a private reservation value v_i that is an independent draw from a distribution F_v with support $V=[0,v^u]$. A bidder with value v_i who submits a bid b_i receives a monetary payment of $w_i=v_i-b_i$ if she wins the auction, i.e., if $b_i>b_j$ for all $j\neq i$ and receives $w_i=0$ otherwise. In the CRRAM model, bidders are assumed to display monetary payment utility functions of the form $U(w,r)=w^r$. The coefficient r is the Arrow-Pratt index of constant relative risk aversion. We assume that r is distributed across individuals according to some distribution function G_r with support $R=[0,\bar{r}]$. The distributions F_v , G_r and the number of bidders N are common knowledge, but the value realizations v_i and the coefficient or risk aversion r_i are information private to the individual.

In this context, a bidding strategy $b(v,r)$ is a Symmetric Bayes Nash Equilibrium (SBNE) if for all valuations, bidding $b_i=b(v_i,r_i)$ is a best response for bidder i when all bidders $j\leq i$ also bid $b(v_j,r_j)$. Eric Maskin and John Riley (2000) show that if b_i is a bidder's best response, then it is monotone increasing in valuations. Thus, let $p(b,r)$ stand for the inverse of $b(v,r)$. The expected payoff of bidder i is defined as

$$\Pi(b_i;v_i,r_i)=(v_i-b_i)^{r_i} E_r[F_v(p(b_i,r))]^{N-1} \quad (1)$$

If an individual with valuation v_i is planning to bid a quantity b_i , then the quantity $q_i=(v_i-b_i)/b_i$ is the monetary profit per dollar bid in case of winning the auction with b_i . In equilibrium, there exists a relationship between the profit per dollar, the risk aversion coefficient, and the probabilities of winning, arising from the first-order conditions of maximization of (1):

$$r_i = q_i \frac{\partial \ln E_r[F_v(p(b,r))]^{N-1}}{\partial \ln b} \quad (2)$$

The above equation implies that in equilibrium the coefficient of risk aversion must equal the profit per dollar at stake times the elasticity of the chances of winning. This elasticity can be obtained in closed form if we allow F_v to take a specific parametric form. The data set to be used in the next section comes from auction experiments that use the uniform distribution to sample individual valuations. Without loss of generality, normalize the support of the uniform to $[0,1]$. Then, the chances of winning with a bid of b simplify to $E_r p(b,r)^{N-1}$ whenever $b \leq \bar{b}$ where \bar{b} is the maximum bid of the least risk-averse bidder, i.e., $p(\bar{b}, \bar{r})=1$. In that case, it can be shown² that the differential equation in (2) has a closed form solution with

$$b(v_i, r_i) = \frac{N-1}{N-1+r_i} v_i \quad \text{for all } v_i \leq \frac{N-1+r_i}{N-1+\bar{r}} \quad (3)$$

After substitution in (2), it is found that the expression for the profit per dollar reduces to $q_i=r_i/(N-1)$. We can also use (3) to find the bidding function for the particular case where subjects share the same risk aversion coefficient r . The risk-averse symmetric Nash equilibrium (RASNE) is

$$b(v_i) = \frac{N-1}{N-1+r} v_i \quad (4)$$

for all v_i , while the risk-neutral Nash equilibrium reduces to

$$b(v_i) = \frac{N-1}{N} v_i \quad \text{for all } v_i \in [0,1] \quad (5)$$

2. A First Empirical Analysis of the Data

Dyer, Kagel, and Levin (1989) (DKL) investigate the effect of uncertainty about the number of bidders³ on the market outcomes of first-price sealed-bid auctions with independent private values. They design a series of experiments with subjects recruited from MBA classes at the University of Houston. There are three sessions. In each session, a different group of six subjects act as bidders competing for the possession of a single item. Sessions are divided into a series of trading periods, each period consisting of either two small ($N=3$) markets or one single large ($N=6$) market. Our first source of data corresponds to the DKL data where we have available 23 periods or rounds for the first two groups of six bidders and 19 rounds for the last group. This gives 780 observations, half of them with $N=3$ and half with $N=6$ bidders.

² A detailed guide on how to invert the bidding function and solve for equilibrium can be found in Cox, Smith, and Walker (1988) or Maskin and Riley (2000).

³ Because the purpose of the Dyer, Kagel, and Levin (1989) experiment is different from what is intended here, we selected from their experimental design only those sessions where there was no uncertainty on the number of bidders named by the authors as the "contingent" sessions.

The second source of data is taken from Cox, Smith, and Walker (1988) (CSW). These authors collect data from eight series of experiments of sealed-bid first-price and Dutch auctions in Cox, Roberson, and Smith (1982) and in Cox, Smith, and Walker (1985). We select only those series of experiments consisting of sequences of the first-price, named series 4 in the CSW paper, and consisting of 10 sessions of 25 rounds or auctions with 4 bidders. Valuations are drawn uniformly from $[0,10]$.

Summarizing, the data set consists of 1780 observations, 390 with $N=3$ (DKL3), 1000 with $N=4$ (CSW4), and 390 with $N=6$ (DKL6) bidders. For each of the 40 individuals and round, bids and valuation are observed.

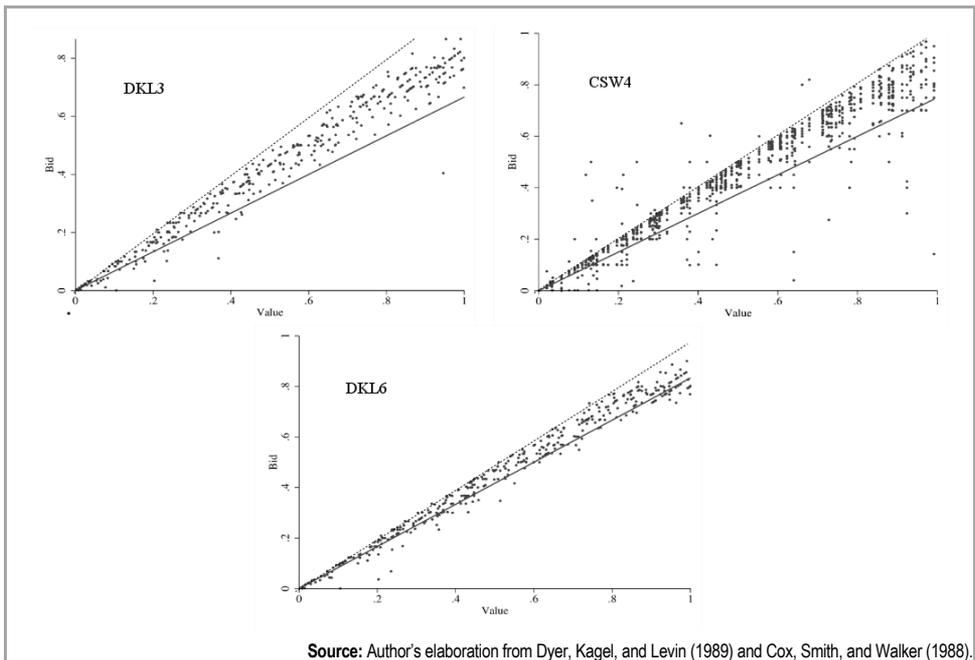


Figure 1 Plot of Bid Data; Solid Line is the RNE Prediction; Dashed Line is the 45° Degree

Figure 1 shows a scatter plot of bids and valuations for the $N=3$, $N=4$, and $N=6$ cases labeled as DKL3, CSW4, and DKL6, respectively. The risk-neutral equilibrium prediction as of equation (6) and the 45-degree line are also plotted. Noticeable is the pervasive presence of bids above the risk-neutral prediction (*overbidding*), particularly above the 0.5 valuations. This phenomenon is more extreme in CSW4 where some bids are above the corresponding valuations – the 45 degree line – that do not appear in the CSW data and may reflect differences in the experiment design. Also in this panel, we can find extremely low bids for low-medium valuations. For the data in DKL3 and DKL6, bids are much less dispersed, especially in the latter. The differences in the bid spread, i.e., the distance between bid and the 45 line of valuations, are another interesting aspect of the data: for valuations above one half,

the bid spread seems to be non-monotonically increasing, particularly in the DKL3 and DKL6 cases, contrary to the linear bidding prediction of the equilibrium model.⁴ Moreover, this phenomenon seems less evident for CSW4, pointing again to substantial differences in the outcome of both experiment designs.

We now propose a model for the CRRAM equilibrium bidding using the profit per dollar variable (q), instead of the raw bids. As shown in Section 2, under equilibrium and uniform valuations⁵ the profit per dollar q_{it} of bidder i in round t should be constant and equal to $r_i/(N-1)$, thus not depending on valuations. Hence, our first specification for q_{it} includes a constant term and a quadratic function on valuations. Individual (c_i) and time (w_t) effects are also included to capture unobserved heterogeneity distributed across agents or across rounds t :

$$q_{it} = \gamma_0 + \gamma_1 v_{it} + \gamma_2 v_{it}^2 + c_i + w_t + u_{it}. \quad (6)$$

Significance of the γ_1 , γ_2 coefficients will be read as a rejection of the CRRAM model. Otherwise, an estimate of the individual coefficient of risk aversion may be obtained as $(N-1)(\gamma_0+c_i)$. Table 1 shows the results of estimating (6) using different specifications. The third column shows the estimates of pooled ordinary least squares (OLS) of (6) without including individual and time effects. The second column shows the estimation of a model with random effects c_i and including time dummies w_t . The third column presents the estimates of a random effects model in which variables are in deviation with respect to the mean of the corresponding round t , to account for the time effect.

Table 1 Estimates of (6); Entries in Italics Indicate Non-significance of the Coefficient at 1%

Number of bidders	Coefficient	Random effects		
		OLS	Round dummies	Deviations
3	γ_0	0.298	0.404	0.380
	γ_1	-0.314	-0.612	-0.557
	γ_2	0.260	0.486	0.448
4	γ_0	0.188	0.336	0.187
	γ_1	-0.397	-0.380	-0.386
	γ_2	0.421	0.416	0.411
6	γ_0	0.253	0.315	0.264
	γ_1	-0.464	-0.507	-0.495
	γ_2	0.402	0.431	0.422

Source: Author's calculations from Dyer, Kagel, and Levin (1989) and Cox, Smith, and Walker (1988).

⁴ Engelbrecht-Wiggans and Katok (2007) consider the role of the money that the winner ceases to earn once she is aware that the second bidder's bid is below hers (MLOT regret) and the missed opportunity to win if the second bidder's bid is above price (MOTW regret). If bidders do care about MLOT and MOTW regret, the observed non-linearity could well respond to a higher expected amount of MLOT regret or a smaller amount of MOTW regret, or both.

⁵ As pointed out by one referee, the equilibrium is not unique and other non-linear equilibria may exist though they are harder to characterize. Thus, rejection of linear bidding does not necessarily mean rejection of the equilibrium model as a whole.

Estimates of the γ_1, γ_2 coefficients are found to be significant in almost all cases, except for the OLS and $N=3$, which may respond to the relative inefficiency of OLS with respect to generalized least squares in panel estimation. It is worth noting that the negative sign on the coefficient γ_1 does not imply that q is decreasing on valuations, because the specification is quadratic. In fact, if we consider the γ_2 coefficients, the profit per dollar invested q is increasing in valuations. Since q is defined as $(v-b)/b$, the mean bid b/v must be decreasing or, equivalently, the bidding function must be concave.

A Wald test on the significance of the w_t round effects shows significance at least for the CSW4 ($p < 0.05$) and DKL6 ($p < 0.01$) cases. Moreover, a Breusch-Pagan Lagrange multiplier test for the individual effects shows significance ($\text{Prob } \chi^2(1) < 0.01$) in all cases. The individual differences account for nearly one quarter of the unobserved variability. Therefore, the variability *between* individuals is relevant to explain the bidding behavior in our data set.

The model in (6) incorporates individual effects but ignores the presence of variability *within* individuals. The presence of individual effects assumes that bidding behavior may be clustered at the individual level alone so that we impose the group structure of the data, while it would be better to relax such a restriction by letting the data themselves tell us the best characterization of the data in terms of clusters. This is the main purpose of clustering analysis, and what we aim to do in the next section.

3. The Model-Based Clustering Approach

In the analysis of clusters (say, clustering) observations are classified into groups of homogeneous data that are similar according to the criterion and measure of similarity chosen. In this paper, we follow a model-based approach to clustering (MBC). To our knowledge, MBC had never been used before in the context of empirical auctions or experimental data. In MBC a flexible probabilistic model such as a mixture of densities characterizes the data-generating process. Once estimated, observations are assigned to clusters using the ex-post probability of sampling from each of the mixture components. By doing so, we may find homogeneous groups of observations whose characterization may be tested against the Nash predictions or alternatives. Another advantage of MBC is that it permits weighting for the ex-post probability that each observation belongs to that cluster when we are interested in estimating a model for one particular set of observations, for instance those following the Nash predictions. That point, however, is out of the scope of this paper.

Let q represent as before the profit per bid and v the individual valuations. We will assume that q is sampled from one of H different sub-populations with some ex-ante sampling probability given by parameter π_h for $h=1, \dots, H$. The sub-populations are assumed to be normal with unknown mean and variance, and thus characterized by a vector of parameters $\theta_h^i = (\gamma_{0h}, \gamma_{1h}, \gamma_{2h}, \sigma_h)$.

The mean of cluster h is defined as $E(q | v, \gamma_h) = \gamma_{0h} + \gamma_{1h}v + \gamma_{2h}v^2$, where v are the individual valuations.⁶ Finding that the γ_{1h}, γ_{2h} are non-significantly different from zero for one particular h would imply that the observations sampled from this sub-population have constant mean, or equivalently that the bids were obtained from a linear bidding function, as shown in Section 2. Thus, the observations belonging to this cluster would meet the restrictions that the equilibrium models impose.

How does the MBC assign observations to clusters? The procedure implies maximum-likelihood estimation (MLE) of the parameters of interest by maximization of the normal mixture log-likelihood of the data:

$$\ln \ell(\theta'_1, \pi_1, \dots, \theta'_H, \pi_H) = \sum_{it} \ln \left[\sum_h \pi_h f(q_{it} | v_{it}, \theta'_h) \right] \tag{7}$$

where $f(q | v, \theta'_h)$ is the normal density, with the mean and variance parameterized with θ .

The MBC is in essence a problem of latent variables, where the researcher can observe the data point but not the cluster it belongs to. In this context, the EM algorithm (Geoffrey McLachlan and David Peel 2000) is particularly useful in finding maximum likelihood estimates of the parameters of interest θ in (8). The EM algorithm performs two steps at every iteration: in the first step (M-step), estimates $\hat{\theta}_h$ for $h=1, \dots, H$ are obtained after maximization of the complete log-likelihood of the data and the latent. In the second step, estimates of the posterior probabilities of cluster membership of subject i in period t (say s_{it}^h) are found using

$$s_{it}^h = \frac{\pi_h f(q_{it} | v_{it}, \theta'_h)}{\sum_h \pi_h f(q_{it} | v_{it}, \theta'_h)} \tag{8}$$

The algorithm goes on until a convergence criterion is reached. In this paper, we use Aitken's acceleration procedure (Dankmar Böhning et al. 1994), which improves the speed and numerical stability of the EM algorithm.

After convergence, estimates of the parameters of interest θ and the posterior probabilities s are available. Both are of interest in our research: parameters θ characterize the clusters that generated the data, and posterior probabilities s can be used for cluster classification. Inference on θ makes use of the covariance matrix of the parameters of interest computed with the gradients of the complete log-likelihood and a Parzen kernel as suggested in McLachlan and Peel (2000). Posterior probabilities s

⁶ Note that the individual CRRAM effects disappear from the specification of the mean. The reason is that under the clustering specification, with the underlying data generated according to the CRRAM model, we would find that individual bids belong to one single cluster and that the number of clusters H is the number of different risk aversion coefficients among the individuals. Therefore, the MBC specification of the mean encompasses the CRRAM individual effects.

are used in the classification of the data as follows: we compute \hat{S}_{it}^h for $h=1, \dots, H$ using (8) and the estimates $\hat{\theta}$, and then we assign each observation to the cluster with the highest ex-post probability.

3.1 The Number of Components

Starting from the single sample ($H=1$), we estimate the mixture with up to seven components in search for the best specification. The reason is that deciding the number H of members (or clusters) in the mixture is a difficult issue and an old debate in the literature. Basically there are two approaches to the problem: hypothesis testing versus fit criterion such as the Bayesian Information Criterion (BIC). Because the technical difficulties associated with the likelihood ratio test or other alternatives are burdensome⁷ we will use the second approach. The BIC penalizes the number of parameters by adding two times the number of parameters and the log of sample size to minus the log-likelihood. When the BIC is used as a guide, finding a parsimonious parameterization of the model as an outcome is expected.⁸

Table 2 Bayesian Information Criteria (BIC) of the Mixture for Different Specifications on the Number of Components

Num. of members (H)	Num. of parameters	DKL3	DKL6	CSW
1	3	-429.49	-668.18	-915.64
2	7	-440.04	-812.99	-259.72
3	11	-471.71	-818.04	-464.71
4	15	-470.54	-844.24	-522.47
5	19	-470.61	-838.01	-517.81
6	23	-454.33	-823.72	-509.36
7	27	-455.54	-821.98	-512.15

Source: Author's calculations from Dyer, Kagel, and Levin (1989) and Cox, Smith, and Walker (1988).

Table 2 shows a summary of the BIC for the different specifications. For CSW4 and DKL6, the minimum BIC is reached at $H=4$, while the minimum for DKL is reached at $H=3$, although the differences between the $H=3$, $H=4$, and $H=5$ are small and $H=4$ is a good choice in terms of fit for all three treatments. For this reason, the analysis that follows will be referred to the $H=4$ specification.

3.2 Characterizing the Clusters

Table 3 shows the estimates of the ex-ante probabilities of clusters and the coefficients of the mean of the normal components of the mixture.

Figures 2, 3, and 4 plot the observations classified by cluster where each observation is assigned to the cluster with the highest ex-post probability.

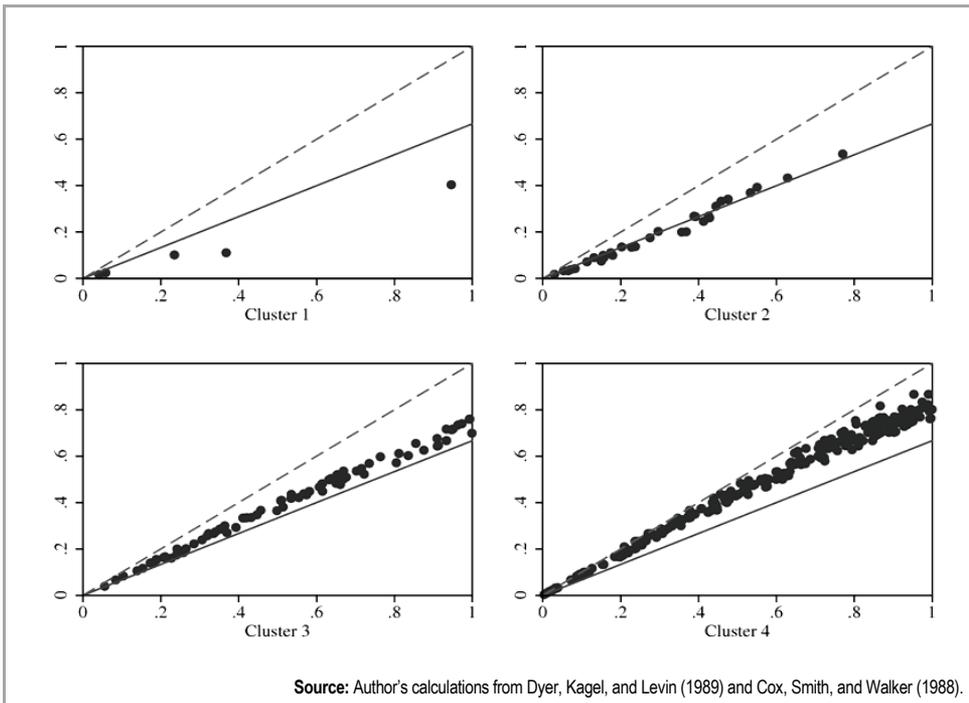
⁷ See, for instance, Bruno Goffinet, Patrice Losiel, and Beatrice Laurent (1992), Jiahua Chen and Ping Cheng (1997), or Didier Dacunha-Castelle and Elisabeth Gassiat (1997) for examples of very limited and particular solutions.

⁸ Brian G. Leroux (1992), for instance, proves that the BIC does not underestimate the number of components asymptotically.

Table 3 Estimates of the Mixture Model with 4 Clusters; Entries in Italics Indicate Non-significance at 1%

		Parameters (θ)				Test
Cluster		π	γ_0	γ_1	γ_2	$\gamma_1=\gamma_2=0$
DKL3	1	0.0134	1.1496	<i>3.4491</i>	<i>-3.4076</i>	1.93
	2	0.1576	0.8598	-1.3116	<i>0.7048</i>	24.3
	3	0.2597	0.2814	<i>-0.1342</i>	<i>0.1398</i>	0.2
	4	0.5693	0.1052	<i>0.0107</i>	0.1061	15.4
CSW4	1	0.1439	0.4234	<i>-0.6632</i>	<i>0.7792</i>	1.34
	2	0.1585	<i>0.0049</i>	<i>0.0315</i>	<i>-0.0114</i>	19.6
	3	0.3290	0.0663	-0.1667	0.2233	124.5
	4	0.3686	0.1958	-0.3450	0.4011	43.3
DKL6	1	0.0830	0.3774	-1.1434	1.0219	216.7
	2	0.1227	0.7523	-1.4522	<i>0.9022</i>	81.5
	3	0.1687	0.3032	-0.4019	0.3375	13.6
	4	0.6256	0.1043	-0.1430	0.2032	29.6

Source: Author's calculations from Dyer, Kagel, and Levin (1989) and Cox, Smith, and Walker (1988).



Source: Author's calculations from Dyer, Kagel, and Levin (1989) and Cox, Smith, and Walker (1988).

Figure 2 DKL3 Observations Classified by Cluster

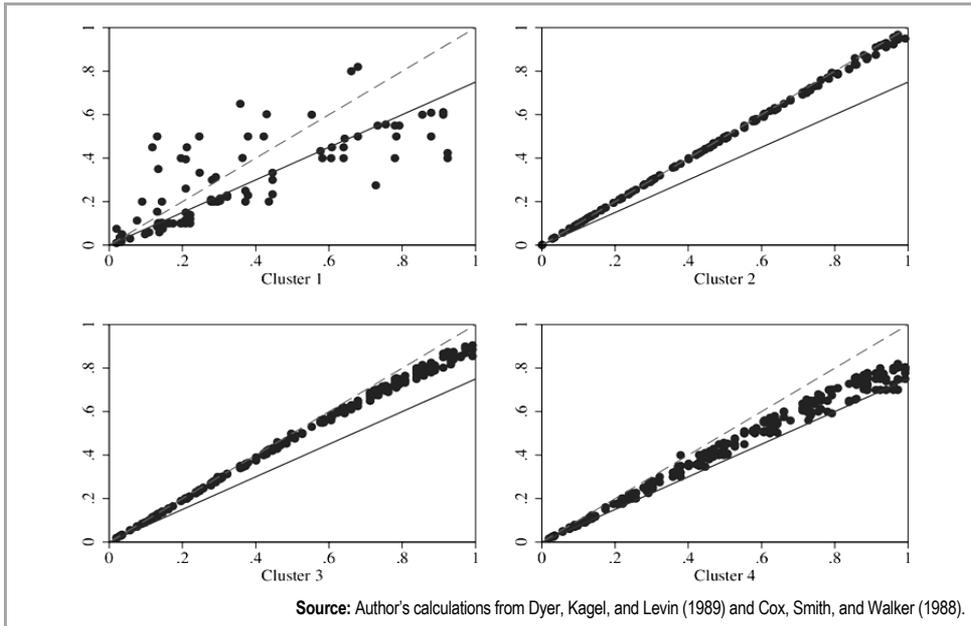


Figure 3 CSW4 Observations by Cluster

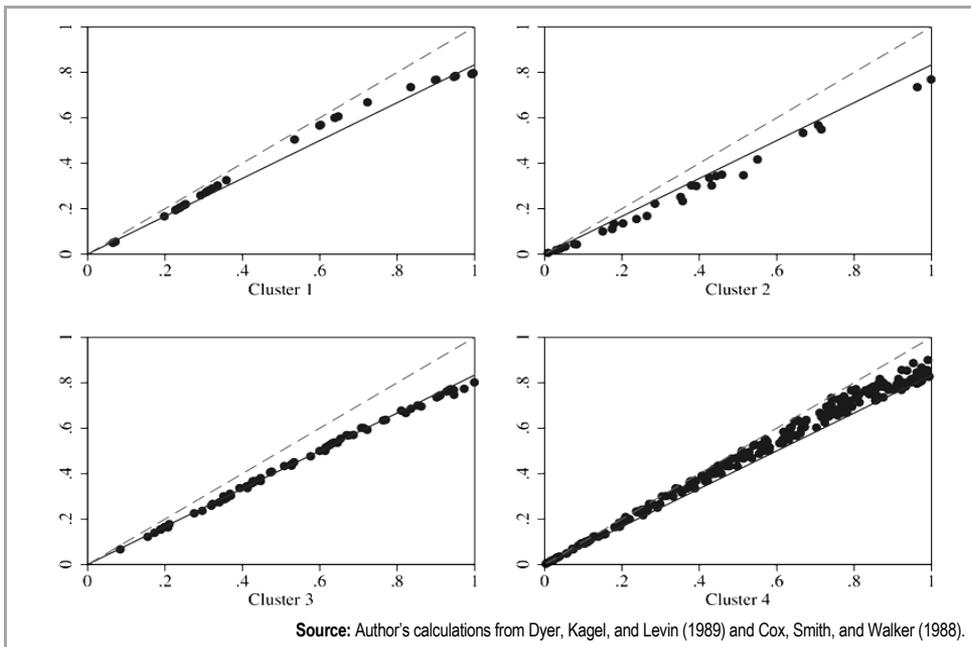


Figure 4 DKL6 Observations by Cluster

Our main conclusions are as follows:

Clusters differ in their relative size. Thus, clusters 3 and 4 contain more than 80% of the sample in DKL3, more than 70% in CSW4, and almost 80% in DKL6. Therefore, it seems that there are some bidding strategies that are particularly popular among agents. It is found that in particular the biggest clusters are characterized by *overbidding* plus non-linearly concave bidding behavior.

Clusters with the highest size show non-linear bidding and thus reject the equilibrium model. In all treatments, the type of bidding classified by these high-frequency clusters is similar. In CSW4, bids in clusters 3 and 4 are characterized by non-linear bidding, as confirmed by the individual and joint significance of the linear and quadratic coefficients in Table 3. In DKL6, clusters 3 and 4 also represent non-linear bid functions, although the dispersion in general for these data is much smaller than in other treatments, thus increasing the precision of the tests. Visual inspection of cluster 3 in Figure 3, for instance, shows a slight downward slope at the 0.8 valuation and above, enough to reject the hypothesis of linear bidding as shown in Table 3. A similar phenomenon appears in cluster 2 of DKL3 where the presence of a few “underbidding” observations leads to rejection of linear bids. Finally, clusters 3 and 4 in DKL3 differ in that cluster 3, which contains 26% of the sample, does not reject the risk-averse equilibrium model of Section 2. However, the CRRAM model is rejected for cluster 4 with 57% of the sample.

Some clusters capture noisy or risk-loving bidding. This is quite evident in the case of cluster 1 of DKL3 and CSW4 data. In the first case, an estimate of $\gamma_0=1.15$ would mean under CRRAM a risk aversion coefficient of 3. In other words, this cluster is capturing risk-loving subjects betting to earn a high profit per dollar exposed despite the risk. The upper left panel of Figure 3 shows the classified observations for this cluster position to be well below the RNE prediction. This behavior is nevertheless very minority with less than 1.5% of prior. This is not so much for cluster 1 in the CSW4 data. The type of behavior captured in this cluster is extremely noisy and diffuse (see first panel of Figure 3). An important number of *irrational* or *dominated* bids in this cluster are even above the 45-degree line, i.e., above self-valuation and with negative expected profit, and this makes a difference with the DKL3 case, which makes us consider the role of experiment design in obtaining this noisy behavior. It is interesting to note the ability of the clustering methodology to identify correctly those observations and to classify them in an own cluster.

A significant part of the heterogeneity is due to individual differences. Do individuals tend to bid persistently as of one particular cluster? We are interested in checking whether the clustering process has attributed individual behavior to particular clusters, thus separating the agents. To this purpose, we build a two-way table of the observations attending to individual and cluster, and we compute some statistics as shown in Table 4.

Column 1 shows the ex-ante probability for each cluster for reference. Column 2 displays the average ex-post probability (see the definition of s^h in equation 8) of each cluster computed over the individuals whose bids are classified as belonging to that cluster more (less in column 3) frequently than expected by π . The number of individuals so classified is given in parenthesis. Roughly, the numbers in columns 2

Table 4 Summary of Results of a Two-way Table of Cluster and Individuals; All Tests are Significant at 1%

	Cluster	Ex-ante	Mean posterior		Pearson test
		Prob (π)	Above	Below	
DKL3	1	0.0134	0.06(4)	0.00(14)	164.06
	2	0.1576	0.23(6)	0.02(12)	
	3	0.2597	0.41(7)	0.11(11)	
	4	0.5693	0.88(9)	0.46(9)	
CSW4	1	0.1439	0.26(15)	0.02(25)	650.03
	2	0.1585	0.46(12)	0.07(28)	
	3	0.3290	0.58(14)	0.23(26)	
	4	0.3686	0.58(17)	0.17(23)	
DKL6	1	0.0830	0.17(9)	0.02(9)	144.52
	2	0.1227	0.21(5)	0.03(13)	
	3	0.1687	0.28(12)	0.01(6)	
	4	0.6256	0.83(8)	0.48(10)	

Source: Author's calculations from Dyer, Kagel, and Levin (1989) and Cox, Smith, and Walker (1988).

and 3 give an idea of how many bidders would be or would not be labeled as “cluster X bidders” and for how much. Thus, for cluster 1 in the DKL3 data, 4 out of the 18 individuals in the sample send more than the ex-ante expected 1.4% of their bids as cluster-1-type bids. For those *more-than-the-rest* cluster 1 bidders the average ex-post probability of cluster-1-type bids is around 6%. Cluster 1 in DKL3 depicts a risk-loving compatible behavior, so that it could be said that there are 4 individuals in the sample that occasionally (6%) bid risky, while the remaining 14 never try that. On the other hand, one half of the individuals in DKL3 (9 out of the 18) tend to bid as of cluster 4, i.e., the concave bidding above the RNE prediction type 90% of the time. Something similar happens in CSW4 where 17 out of 40 subjects bid 58% of times according to cluster 4. It must be noted here, however, that 15 out of 40 bid as of cluster 1, a cluster with dominated bids and very noisy, more than 25% of the time. Finally, DKL6 also shows that almost one half of the subjects tend to bid concave and above RNE 83% of the time (cluster 4 of DKL6).

Finally, the last column of Table 4 presents a Pearson test of independence between clusters and subjects. The main conclusion is that individual differences explain a significant portion of the cluster classification in all datasets but that the relation is far from perfect. As an illustration, the bids of individual number 2 of CSW4 do not seem to respond to any particular pattern: she bids 6 times as of cluster 1, 4 times as of cluster 2, 8 times as of cluster 3, and 6 times as of cluster 4, so that her behavior cannot be easily rationalized into one consistent bidding strategy.

4. Conclusions

In this paper, we have revisited laboratory auction data aiming to the prevalence of non-linear bidding behavior and identifying noisy or irrational behavior that could be responsible for the observed data dispersion in experiments. When we fit a mixture model to the data, we obtain several interesting findings: that the clusters of bigger size represent non-linear concave bidding, that noisy or risk-loving attitudes are filtered in one cluster, and that a significant portion of the clustering is explained by individual behavior. Regarding this last point, however, it is also noted that clusters do not exactly classify individuals. In other words, most individuals display bids of different clusters. This is consistent with a framework where experiment subjects do not consistently follow a bidding strategy but randomize among alternatives or even a framework similar to the approach of Armantier and Treich (2006) where bidders move among a set of different bidding strategies depending on their prospects of winning.

In view of the results, it seems that once we eliminate the mask of noisy or gambling bidding that increases the dispersion of the data, subjects in experiments are not only characterized by the well-known phenomenon of overbidding but also by a persistent trend to display concave bidding functions, which leads us to conclude that more emphasis in explaining this anomaly of the bidding data should be included in the future research agenda.

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