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Liquidity, Welfare and Distribution

Summary: This work presents a dynamic general equilibrium model where wealth distribution is endogenous. I provide channels of causality that suggest a complex relationship between financial markets and the real activity which breaks down the classical dichotomy. As a consequence, the Friedman rule does not hold. In terms of the current events taking place in the world economy, this paper provides a rationale to advert against the perils of an economy satiated with liquidity. Efficiency and distribution cannot thus be considered as separate attributes once we account for the interactions between financial markets and the economic performance.

Key words: Friedman rule, Idiosyncratic risk, Liquidity.

JEL: E22, E31, E40.

The current crisis presents evidence against the convenience of a market economy satiated with liquidity. The effects of the financial factors over the business cycle have been studied extensively in the literature. This class of models (Mark Gertler and Nobuhiro Kiyotaki 2010) introduces an agency problem between ownership and control which establishes a wedge between the external and the internal cost of finance. As a result, the balance sheet is a key determinant of the cost of credit, giving rise to a positive feedback between the financial and the real sector (financial accelerator).

However, this body of the literature omits an explicit role for liquidity, which seems to be a cause of the financial turmoil. Milton Friedman (1969) argued heuristically that an economy cannot be efficient unless agents do not economize on money balances. Consequently, the monetary authorities should deflate prices at a fixed rate so the nominal interest rate reaches as low as possible. Friedman's article prompted an immediate reaction by some economists. Miguel Sidrauski (1967), William A. Brock (1974; 1975) and Truman Bewley (1977; 1980) among others, confirmed the optimality of the Friedman rule (FR). By contrast, under the assumption of alternative distortionary taxes, Edmund S. Phelps (1973) concluded that in a Ramsey equilibrium, the optimal inflation tax is positive and equals the deadweight loss of alternative taxes. Robert Lucas and Nancy Stokey (1983), Kent P. Krimbourgh (1986), Chari V. Varadarajan, Christiano J. Lawrence, and Patrick J. Kehoe (1991), Pablo Guidotti and Carlos A. Veigh (1993), suggested that the presence of distortionary taxes does not imply a positive optimal inflation tax.

In this article I show that the FR is suboptimal in a stochastic dynamic general equilibrium model in an environment isomorphic to Friedman's (1969). Households are assumed to face idiosyncratic income shocks which can be partially buffered by holding capital and inconvertible money. Incomplete market models with idiosyn-

cratic risk were first introduced by Bewley (1977; 1980; 1983) to formalize classical issues such as the permanent income hypothesis and the FR itself, as noted above. In the same spirit, Rao S. Aiyagari (1994), Mark Huggett (1997) and Tiago V. de V. Cavalcanti and Anne P. Villamil (2003) addressed the capital accumulation patterns and the determination of the interest rate under the presence of idiosyncratic uncertainty and showed the existence of a precautionary motive of savings. Per Krusell and Anthony A. Smith (1998) showed that in a Bewley economy the aggregate dynamics are mostly explained by the first moment of wealth distribution. In contrast, this article provides an example where idiosyncratic uncertainty matters substantially in terms of policy implications. This paper provides a rationale to avert from the perils of an economy satiated with liquidity. Efficiency and distribution cannot thus be considered as epistemically separable attributes once we account for the interactions between financial markets and the economic performance. Ayse İmrohoroğlu (1992) and Andrés Erosa and Gustavo Ventura (2002) have studied the distributional impact of inflation to criticize the optimality of the FR. However, the analysis of these articles is limited to the study of stationary equilibria: in the standard Cass-Koopmans one sector-growth model, the golden rule is not a valid criterion for utility maximization. Consequently, the transition path turns out to be a relevant factor as far as welfare is concerned.

Market incompleteness has been proven to be a key issue to determine the optimum quantity of money. In Timothy Kehoe, David Levine, and Michael Woodford (1992), the distribution of real balances is degenerate. They set up a stochastic dynamic general equilibrium model where the government issues nominal non-state contingency liabilities. For this reason, price volatility has the potential to absorb unexpected movements in fiscal expenditures. As Stephanie Schmitt-Grohé and Martín Uribe (2004) have shown, a small degree of price stickiness under imperfect competition induces a deviation of the FR. Aubhik Khan, Robert G. King, and Alexander L. Wolman (2003) and Nicola Acocella, Giovanni di Bartolomeo, and Alexander L. Wolman (2011) reach similar conclusions. Inflation plays the role of an indirect tax on profits whenever the government cannot undo market distortion through fiscal instruments.

Other examples where an expansionary monetary policy dominates the FR are provided by Edward Green and Ruilin Zhou (2002), Perry Mehrling (1995), Beatrix Paal and Bruce D. Smith (2000), Alexei Deviatov and Neil Wallace (2001) and Smith (2002). İmrohoroğlu (1992) computed the inflation cost in a pure monetary economy similar to the model herein presented. She found that the welfare costs of inflation are higher than those reported by Martin Bailey (1956). The analysis of this paper differs from İmrohoroğlu (1992) and Kehoe, Levine and Woodford (1992) in that the minimal amount of income is here very small, a fact that exacerbates the distributional issues.

In the next two sections I offer the description of the environment. I point to distribution as a more remote variable to consider than market incompleteness, imperfect competition and sluggish price adjustment, to configure the optimal monetary policy. Section 3 defines the competitive equilibrium, Section 4 presents the main results and Section 5 concludes. An appendix provides the proof of Proposition 1.

1. The Model

The economy is populated with households, competitive firms producing a homogeneous commodity and a monetary authority holding the monopoly right to issue money. Each period, consumers suffer an idiosyncratic income shock which originates an endogenous dynamic of wealth distribution. Since this is the only source of uncertainty, the aggregates evolve deterministically under perfect foresight (Kenneth L. Judd 1985). There is no market for borrowing and lending, in line with Bewley (1980), Aiyagari (1994) and many others.

In Guillermo Calvo and Pablo Guidotti (1993) and Chari, Lawrence, and Kehoe (1991) it is (Ramsey) optimal to use unanticipated inflation tax in a stochastic environment with distortionary taxes and nominal public debt. Schmitt-Grohé and Uribe (2004) have shown that under slight price rigidities and imperfect competition it is optimal to promote price stability. Since our model is a reduced form of a more complex set of relations, I will assume that money is issued by the government at a fixed rate

$$M_{t+1}^S = (1 + \sigma)M_t^S,$$

M_{t+1}^S stands for *nominal* supply of money. If σ is positive (negative), money is injected (withdrawn) from the public. Money is held because of their transaction services (*liquidity*), captured in the utility function as we shall see later.

The only final, non storable good, c , is produced according to a neoclassical production function given shared by a set of firms operating in a perfectly competitive market. Aggregate output is therefore a function of aggregate capital (K) and total labour (L)

$$Y = F(K, L) - \delta K,$$

where δ is the depreciation rate. Since the total labor is normalized to one, the output can be expressed in terms of the aggregate capital K :

$$f(K) = F(K, 1) - \delta K.$$

The function f satisfies the usual Inada conditions. The optimality conditions are

$$\begin{aligned} r &= f'(K) \\ w &= f(K) - f'(K)K. \end{aligned} \tag{1.1}$$

2. Consumers

2.1 Preferences and Transaction Technology

There is a continuum of households that share identical preferences defined over a stochastic process of consumption and real balances,

$$(1 - \beta)\mathbf{E} \sum_{t=0}^{\infty} \beta^t [u(c_t) + \phi(m_t)], \tag{2.1}$$

with $0 < \beta < 1$ being the rate of preference. Individual real balances are denoted by m . E is the expectation operator and let us assume that expectations are formed rationally. Utility functions (u, ϕ) obey the Inada conditions. Robert C. Feenstra (1986) argued that models with money in the utility function are isomorphic to economies with a transactions technology. I assume that u has a *finite asymptotic exponent* (greater than 1). The exponential utility function fails to hold this property. On the contrary, our assumption rules out the possibility that the difference between the marginal utilities of two close values of c differ greatly. I also assume that u dominates asymptotically to ϕ which implies that the fraction of wealth held in liquid assets decreases with wealth (Erosa and Ventura 2002). These two assumptions can be written in the following way:

$$1 < \lim_{c \rightarrow \infty} -\frac{\ln u'(c)}{\ln c} < \infty \tag{H1}$$

$$\lim_{x \rightarrow \infty} \frac{\phi'(x)}{u'(x)} = 0. \tag{H2}$$

When u and ϕ belong to the class of CRRA utility function, the assumption H2 implies that the coefficient of risk aversion of ϕ is greater than that of u . The assumption that the coefficient of risk aversion is greater than one is necessary to obtain Proposition 1, but it is seemingly in line with the empirical evidence.

2.2 The Budget Constraint

Let P_t be the price level. Let us define:

$$m_{t+1} = \frac{M_{t+1}}{P_t}.$$

Therefore, we have:

$$\frac{M_t}{P_t} = \frac{M_t}{P_{t-1}} \frac{P_{t-1}}{P_t} = (1 + \pi_t)^{-1} m_t.$$

This is the stock of money held by a household multiplied by its gross return. Note that the real return of money is inversely related with the rate of inflation and converges to $-\pi_t$ as the time scale tends to zero (continuous time). Likewise, the resources available for consumption at time t from capital assets are given by $(1 + r_t)k_t$. Labor supply is inelastic, but productivity is idiosyncratic and follows a stable Markov process. It is further assumed that efficiency labor is distributed according to the measure ψ_θ . The distribution ψ_θ stochastically dominates (first-order) $\psi_{\theta'}$ if $\theta > \theta'$. With no loss of generality we can assume that all ψ_θ share identical support $\Theta = [\underline{\theta}, \bar{\theta}]$. Let us define the probability space (Ω, \mathcal{F}, P) , where any element of Ω is a history $\{\theta_t\}$, \mathcal{F} is the infinite product of Borel fields of Θ ; and P is the probability measure naturally inferred from ψ_θ . For the technical details, see Ionescu-Tulcea (1949). At the aggregate level, I assume for convenience that the distribution of θ (λ)

rests at a stationary state. This means that the following condition holds, for any measurable set A in Θ :

$$\lambda(A) = \int_A \psi_\theta \lambda(d\theta).$$

Moreover, the total labor supply is normalized to 1, so that

$$\int \theta d\lambda = 1.$$

The consumers' intertemporal budget constraint, expressed in real terms, is given by:

$$c_t + m_{t+1} + k_{t+1} = (1 + \pi_t)^{-1} m_t + (1 + r_t) k_t + \theta_t w_t + \tau_t. \quad (2.2)$$

Total resources (wealth plus income), expressed in real terms, are equal to the right-hand side of (2.2) and are denoted by x_t , with $x_0 > 0$ given. The government distributes uniformly the inflation tax as lump-sum subsidies (if $\sigma > 0$) or transfers (otherwise). Accordingly,

$$\tau_t = \frac{\sigma M_t^S}{P_t}. \quad (2.3)$$

The Markovian property guarantees that an individual state at period t is fully described by $s_t = (x_t, \theta_t)$. In Samuel Gil Martín (2011) I show that the economy is *globally stable*. Therefore, in the sequel I will consider stationary states, characterized by a constant return $z = ((1 + \sigma)^{-1}, 1 + r)$. The wage rate can be dropped as it is determined by r . The time index can be conveniently dropped.

2.3 A Recursive Formulation

The consumer problem expressed in (2.1) and (2.2) are conveniently represented in a *recursive* formulation. Let $a = (m, k)$ and let

$$Z = \{((1 + \sigma)^{-1}, 1 + r) : (1 + \sigma)^{-1} \leq \beta \leq 1 + r\}.$$

The set of admissible z is in the interior of Z ($\text{int}Z$). First, it is convenient to define utility in terms of the variables of state and control (s and a): let $U(x, a) = u(x - a^+) + \phi(m)$, where $a^+ = m + k$. Denoting the forward operator by $'$, wealth evolves according to

$$x' = a \cdot z + \theta w + \tau.$$

The Bellman equation can be written in the following way for $s = (x, \theta)$:

$$v(s, z) = \max_{a \geq 0} \{U(x, a) + \beta \int v(a \cdot z + \theta w + \tau, b, z) \psi_\theta(db)\} \quad (2.4)$$

As proven in Lerma O. Hernández and Jean-Bernard Lasserre (1996), there is a *unique* value function v satisfying (2.4). The optimal decision rules are *unique* and *continuous*. The value function v is strictly increasing, strictly concave and continuously differentiable in x . From the maximum theorem, v varies continuously with θ

and z . The functions $c(s, z), a(s, z)$, with $a = (m, k)$, are the optimal decision rules. The individual wealth, once the optimal rule is applied, $x' \stackrel{\text{def}}{=} x_z'$, evolves according to:

$$x_z' = a(x, z) \cdot z + \theta w + \tau. \tag{2.5}$$

As pointed out by Bewley (1983), there are negative inflation rates that can fail to be *implementable*. In such a case, a positive mass of consumers would be unable to pay the tax necessary to implement the policy. To understand this point it is convenient to define the difference between money balances with respect to the aggregate level,

$$b_t = m_t - \frac{M_t^S}{P_t},$$

and rewrite the budget constraint (2.2) in terms of b_t ,

$$\begin{aligned} c_t + b_{t+1} + k_{t+1} &= (1 + \pi_t)^{-1} b_t + (1 + r_t) k_t + \theta_t w_t \equiv \hat{x}_t \\ \hat{x}_t &= x_t - (1 + \sigma) \frac{M_t^S}{P_t}. \end{aligned} \tag{2.2'}$$

Here I assume that the sequence of gross returns,

$$z_t = ((1 + \pi_t)^{-1}, (1 + r_t)),$$

converges within the intZ. Therefore, the implementability of a deflation rate guarantees the existence of a plan (b_{t+1}^0, k_{t+1}^0) such that,

$$\hat{x}_0 + \theta \sum_{n=1}^t w_n R_n^{-1} \geq \sum_{n=1}^t \frac{i_n}{1 + i_n} b_n R_n^{-1} + R_t^{-1} (b_{t+1}^0 + k_{t+1}^0). \tag{2.6}$$

where R_t is the discount factor $\prod_{n=1}^t (1 + r_n)$ and i , the nominal interest rate. Equation (2.6) indicates that the present value of the future stream of *sure* income up to t plus the initial wealth must be greater than the present value of the costs of holding money. A deflation rate $\sigma < 0$, is said to be implementable when (2.6) is satisfied by all consumers (a.s) at $t = 0$. In a stationary equilibrium, the wage can be written as w_z because it can be inferred from r . Implementability then boils down to

$$b_t \geq \frac{1 + \sigma}{\sigma} \theta w_z. \tag{2.7}$$

The right hand side of (2.7) the present value of the guaranteed labor income is the *natural debt limit* because the ratio $\sigma(1 + \sigma)^{-1}$ is the interest rate paid by money. Keeping this level of real balances avoids the risk of default.

Bewley (1980, 1983) assumed that marginal utility is finite for any level of consumption. Our assumption of infinite marginal utility of zero consumption tends to zero guarantees that any inflation rate close enough to the discount rate (FR) is implementable whenever households have enough wealth to pay for the inflation tax. When households do not hoard enough assets to pay the tax, they risk zero consumption in finite time with positive probability if they suffer a run of adverse shocks,

which can never be an optimal strategy. Conversely, there is always an implementable negative inflation tax when initial wealth is strictly positive (for a positive mass of consumers). Once the economy is near the stationary equilibrium, it is positive to reduce further the inflation tax because households must accumulate more assets to pay for future taxes. This means that households will increasingly hoard assets to pay future taxes (so avoiding the risk of consuming zero in finite time) as deflation increases. By this means, the FR becomes implementable even though it may take long to reach such a rule.

3. Equilibrium

The Euler Equations of (2.4) in steady state are:

$$\begin{aligned} u'(c_t) &\geq \beta\sigma^{-1} \left[\phi'(m_t) + \int u'(c_{t+1}) d\psi_\theta \right] \\ u'(c_t) &\geq \beta(1+r) \int u'(c_{t+1}) d\psi_\theta \end{aligned} \quad (3.1)$$

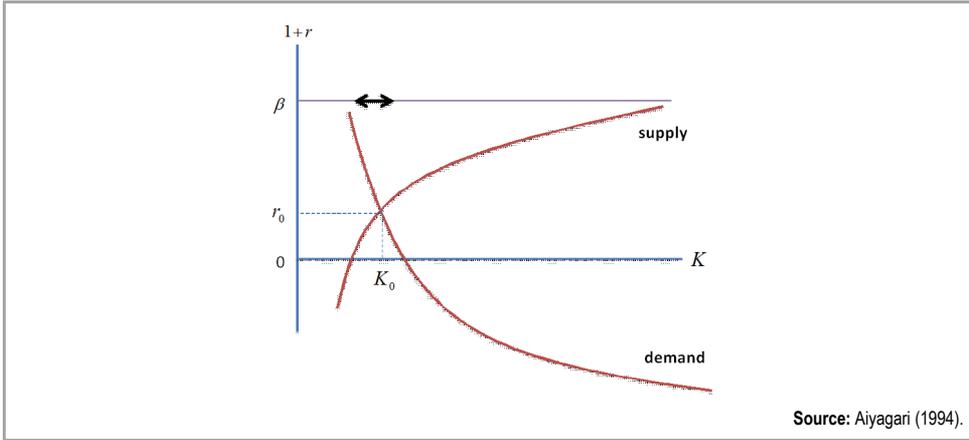
Conditions (3.1) hold with equality when (2.7) is not binding and $k_{t+1} > 0$, respectively. When $\sigma > 0$, equation (3.1) always holds with equality because there is no risk of default.

The properties of the optimal plans are commensurate to the regularity conditions of the fundamentals. The optimal plan varies *continuously* with x : both consumption and assets increase with x and they are *normal*. Both the ratio of money to capital and the ratio of money to consumption decrease with wealth. This implies that inflation is a *regressive tax*.

When any rate of return greater than or equal the rate of discount, consumer resources would infinitely diverge. (See Jack Schechtman and Vera L. S. Escudero (1977) and Marilda A. de Oliveira Sotomayor (1984)). I focus on policy rules such that $1 + \sigma > \beta$.

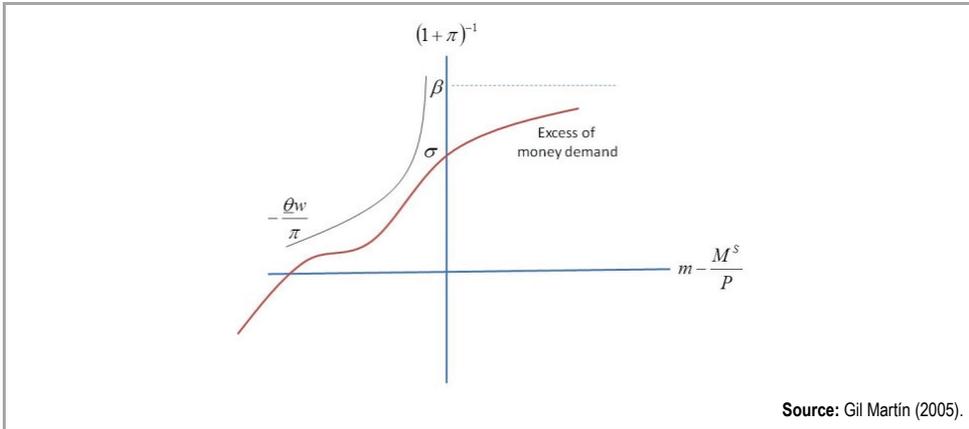
A **Stationary Perfect-Foresight Equilibrium** is defined as a set of *optimal rules*, *value functions* and a *price system* $(a_\sigma, v_\sigma, z_\sigma)$ together with a *distribution* μ_σ of states $s = (x, \theta)$, such that (1) consumers (3.1) and firms (2.1) are rational, (2) markets clear, (3) government budget is balanced and (4) the μ_σ follows an ergodic process consistent with (2.5).

Existence is proven geometrically. There may be several equilibria since the supply of loanable funds (Figure 1) is not necessarily monotonic, as argued in Aiyagari (1994). This is so because a raise in the interest rate, despite having a positive income effect, bids down wages. Uniqueness is guaranteed in a vicinity of β because the capital supply becomes infinitely elastic (Figure 1). The initial condition x , together with θ_t , will determine the optimal decision plans, which in turn uniquely determine the asset returns z_t .



Source: Aiyagari (1994).

Figure 1 Market of Loanable Funds



Source: Gil Martín (2005).

Figure 2 Market of Money

A *monetary equilibrium* is defined formally by $E_\sigma = (a_\sigma, v_\sigma, \mu_\sigma)$. Figures 1 and 2 show the existence of a stationary equilibrium. In terms of welfare, the transition path matters (David Cass 1965). For this reason, we are also interested in defining the non-stationary equilibrium parameterized by an initial distribution of states λ .

$$E_\sigma^\lambda = (a_\sigma^\lambda, v_\sigma^\lambda, \mu_\sigma^\lambda),$$

consisting of sequences converging to a stationary equilibrium E_σ . In the case of μ_σ^λ , the convergence is *weak*.

Remark:

- When $\psi_\theta = 1$ (representative agent), the model is that of Sidrauski (1967). It is easy to prove that the FR is optimal.

- When $\phi = 0$, the model is that of Aiyagari (1994). The stationary equilibrium is represented geometrically by means of the Figure 2.

4. Welfare, Capital Accumulation and Optimal Policy

In a non stationary state, the sequence of optimal plans, resources and welfare can be written as a random variable defined in the product probability space

$$(R_+ \times \Omega, \mathcal{B}(R_+) \times \mathcal{F}, \lambda \otimes P).$$

The σ -fields (information sets) $\mathcal{B}(R_+) \times \mathcal{F}_t = \mathcal{B}(R_+) \times \mathcal{B}(\Theta)^t$ make up a filtration. \mathcal{F}_t is the σ -field generated by the histories θ_n $n \leq t$ and the initial condition λ .

Ex-ante expected welfare is a random variable defined in this probability space:

$$W_\sigma^\lambda = (1 - \beta) \iint \sum_{t=1}^{\infty} \beta^t [u(c_{\sigma,t}) + \phi(m_{\sigma,t})] d\lambda dP. \quad (4.1)$$

The Ramsey problem

$$\max\{W_\sigma^\lambda : 1 + \sigma \geq \beta\},$$

has a solution, not necessarily unique. The main difficulty is that, in general, the optimal policy depends on the initial distribution. The first result (Proposition 1)¹ shows that (4.1) increases in σ within a neighborhood of $\beta - 1$ (FR).

Proposition 1 *If σ is close enough to the FR, the function W_σ^λ (4.1) is strictly increasing in σ provided that $\underline{\theta}$ is small enough and regardless of the initial condition λ .*

A straightforward corollary drawn from the Proposition is that the result on optimality is robust under alternative welfare criteria whose weights are non decreasing in (x, ω) . The crucial factor that determines the optimal monetary policy is the distribution of the efficiency units of labor (uncertainty). Since the FR fails to smooth the consumption paths, this analysis suggests that the welfare benefits of an expansionary policy are of considerable magnitude.

Note that a change in σ brings about distributive effects. There is a massive accumulation of money balances to avoid a risk of default. This money is indeed idle money which does not enter the circuit of current production. Therefore, in the long-run² the FR brings about a situation which can well be defined as a *liquidity trap*. The proof of Proposition 1 rests upon assumptions H1 and H2 (Section 2). They state that the welfare gains of real balances are of second order compared to the welfare gains achieved from consumption. Letting aside the distributional channel, welfare might be increased by bidding up inflation. Low inflation fosters capital accumulation because inflation is negatively correlated with money return. This phenomenon, known as the Tobin effect, is reversed once inflation reaches a threshold level (*pre-cautionary effect*). Thus, as inflation decreases so that it reaches a value close to the

¹ A similar result with iid shocks has been proven in Gil Martín (2010).

² Indeed, near the FR the stationary state takes longer to reach the steady state.

FR, the portfolio-substitution effect dominates the precautionary effect (liquidity trap) and aggregate consumption and capital grow with the rate of injection of liquid balances.

Proposition 2. *When σ is close enough to the FR, aggregate capital, output and consumption increase in σ . There exists a critical level σ^* such that when $\sigma > \sigma^*$ aggregate capital, output and consumption decrease in σ .*

A formal proof can be found in Gil Martín (2005). The insight is that when inflation grows, the substitution effect becomes less and less important. The speculative motives to hold money are less important as the nominal interest rate grows far beyond zero and money is held for transaction and precautionary purposes.

5. Conclusion: Policy Recommendations

Since 2000, the Fed and the European Central Bank have reduced the interest rate for a number of reasons (Maurice Obstfeld and Kenneth Rogoff 2009). There is sparse evidence of increasing wealth polarization after the collapse of the Bretton Woods Agreements in the first half of the 1970s. This process is multidimensional and operates at different scales – countries, firms, households, sectors – and crucially depends on different aspects of the financial market. In my opinion, the policy adopted by the Fed and the ECB may have worsened the polarization process. This point of view agrees with the Keynesian perspective on uncertainty: when the nominal interest rate is close to zero, amid a process of intense financial innovation, it is reasonable to assume that uncertainty is endogenously determined as a result of an increasing (and most of the time invisible) interconnection through the inflating balance sheets of different economic units. In this model, the accumulation of liquid assets under the FR is a reaction against the risk of default (with severe effects on welfare). These balance sheet effects have been analyzed formally in the financial accelerator literature (Gertler and Kiyotaki 2010) to highlight how the complex relations between the economic system and the financial markets give rise to what may potentially become perverse feedbacks effects as those observed in recent events.

Another strand of the literature, based on nominal and real rigidities, suggests that the optimal inflation rate, though different from the FR, is close to zero. In practical terms, this is no real criticism of the FR since Friedman himself never thought to apply a deflationary rule in a real scenario. This constitutes a further example that supports the idea that new Keynesian economics has embedded much of monetarist thought (Brian Snowdon and Howard R. Vane 2005). Returning to our model, as the FR may be associated to high uncertainty, it is reasonable to assume that an optimal monetary policy is inflationary. A further reason to think that the optimal inflation rate is strictly positive is the possibility of fat tails. If they have been observed in financial series, this evidence suggests that individual uncertainty also displays fat tails and long memory. In this sense, distribution gives a powerful explanatory variable which might be in part responsible for the importance of market rigidities and asymmetric information in terms of policy recommendations.

The financial accelerator combined with the distributional channels of monetary policy put forward here are likely to create a feedback loop especially if the FR, as suggested by Keynes and illustrated in the model developed in this article, can

aggravate the problems derived from increasing uncertainty. The contraction of investment, on the one hand, can have an adverse effect on the balance sheet of different economic units. On the other hand, the financial accelerator could have a multiplicative effect on the reaction of aggregate capital to secular inflation. The interactions between the different parts of the economic system make any effort of calibration spurious.

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Appendix

The optimal decision rule a_t converges pointwise to a_σ . Let the family of functions be defined recursively as follows:

$$N_{t+1}^\theta(x, \theta) = a_{t+1}(x, \theta) \cdot z + \theta w_{t+1} + \tau_{t+1}. \quad (\text{A.1})$$

These functions define a one-to-one mapping between x and θ . In symbols, $g_{t+1}^x(x') = \theta$ if and only if $N_{t+1}^\theta(x) = x'$. By ergodicity, $g_{t+1}^x \rightarrow g$ (independent of the initial state x). The following equality holds:

$$\int c_\sigma d\mu_\sigma = \int c_\sigma \circ g^{-1} d\nu,$$

where ν is a measure depending on the preferences and of ψ_θ , defined on the probability space $(\Theta, \mathcal{B}(\Theta))$.

Proof of Proposition 1

Our claim is to show the existence of a sufficiently small real number ε such that the expected welfare is increasing in the interval $(\beta, \beta + \varepsilon)$. The proof proceeds in several steps. The proposition builds upon the ergodicity property of the distribution of wealth μ_t , which is proven upon a standard fixed point argument (See Gil Martín, 2010). Firstly, we make use of two facts: (1) as $1 + \sigma \rightarrow \beta$, individual consumption differs from labor income by an arbitrarily small amount; and, (2) as shown in Cass (1965), the golden rule level of consumption is strictly greater than Pareto optimal allocation. Let k_β be the average capital when $\psi = 1$ (almost surely). Then,

$$\lim_{\sigma \rightarrow \beta-1} \int k_\sigma d\mu_\sigma = k_\beta.$$

It follows that there exists a number η such that for $1 + \sigma \in (\beta, \beta + \eta)$, the average capital is increasing in σ . Besides, since the golden rule level of consumption is strictly less than the one attained by k_β , there should be another real number η' for which consumption increases in capital, and thereby in σ . In the sequel, let $\varepsilon = \min(\eta, \eta')$. In order to simplify notation, let $X_0: \Theta \rightarrow R$ be the limiting random variable of a generic *equilibrium* variable for an arbitrary $\sigma \in (\beta, \beta + \varepsilon)$. That is, $X_0 = X_\sigma \circ s^{-1}$. Let X_ε be defined likewise for a level of inflation $\sigma + \varepsilon$. Since, as pointed out earlier, average consumption increases in σ , we can write the following inequality:

$$\int c_0 d\nu < \int c_\varepsilon d\nu.$$

For convenience we shall use either the subindex 0 or ε to denote whatever variable in equilibrium correspond to the correlative σ . By Euler equation (4.1), and using the (strict) concavity of the utility function,

$$u(c_\varepsilon) - u(c_0) > \beta(1 + r_\varepsilon)(c_\varepsilon - c_0) \int u'(c_\varepsilon)dv. \tag{A.2}$$

We are now interested in showing that the gains of utility in terms of consumption, as measured by u , dominate the deadweight losses of real balances. Analytically,

$$\int (u(c_\varepsilon) - u(c_0)) dv > \int (\phi(m_\varepsilon) - \phi(m_0)) dv$$

By monotonicity of the consumption plan, the random variable has a finite range. Aggregate capital as well as the wage rate are increasing around the inflation rate. Therefore,

$$\int (u(c_\varepsilon) - u(c_0)) dv > \int u'(c_\varepsilon)(c_\varepsilon - c_0) dv \geq \int u'(c_\varepsilon)dv \int (c_\varepsilon - c_0)dv. \tag{A.3}$$

Now, since ϕ is strictly concave,

$$\phi(m_0) - \phi(m_\varepsilon) > \phi'(m_\varepsilon)(m_0 - m_\varepsilon).$$

Let us define

$$R = \frac{(1 + r_\varepsilon)(1 + \sigma) - 1}{\beta((1 + r_\varepsilon))},$$

$$J = \phi'(m_\varepsilon) - R u'(c_\varepsilon).$$

From the Euler equations,

$$\phi(m_0) - \phi(m_\varepsilon) < (R u'(c_\varepsilon) + J)(m_0 - m_\varepsilon).$$

For any pair random variables X and Y for which $|Y - EY| \leq \gamma$, $EXY \leq EXEY - \gamma \sup|X - EX|$. Setting

$$\gamma_g = \sup \left(g'(c_\varepsilon) - \int g'(c_\varepsilon)dv \right),$$

for $g = u, \phi$, and applying this inequality to the last expression, we obtain:

$$\begin{aligned} & \int (\phi(m_0) - \phi(m_\varepsilon)) dv \\ & < R \int (m_0 - m_\varepsilon)dv \int u'(c_\varepsilon)dv + \sigma^{-1} \gamma_u R \theta |w_0 - w_\varepsilon| \\ & + \int J(\phi(m_0) - \phi(m_\varepsilon)) dv. \end{aligned} \tag{A.4}$$

The last two summands of the last equation are negligible (Gil Martín 2010). Let W_0 and W_ε be the average welfare levels corresponding, respectively, to the inflation rates σ and $\sigma + \varepsilon$. Rearranging A.3 and A.4, the necessary condition for $W_\varepsilon > W_0$ is:

$$\int (c_\varepsilon - c_0)dv > R \int (m_0 - m_\varepsilon)dv.$$

It amounts to showing that $(R = R_\varepsilon)$

$$\lim_{\varepsilon \rightarrow 0} R_\varepsilon \varepsilon^{-1} = 0.$$

This property is a straightforward consequence of the following facts:

$$\lim_{\varepsilon \rightarrow 0} \varepsilon^{-1} \int m_\varepsilon d\nu = \infty;$$

and (from the Euler equation and the asymptotical exponential property of the utility function):

$$\lim_{\varepsilon \rightarrow 0} (\phi'(m_\varepsilon)m_\varepsilon - R_\varepsilon u'(c_\varepsilon)) = 0.$$

The second part of the proposition accounts for the transition. The second part of the proposition accounts for the transition. Assume an initial condition $\lambda = \mu_{\beta+\varepsilon}$ (FR). We need to prove that W_σ^λ is locally increasing in σ . That is, starting from a steady state around the FR, raising the inflation rate brings about an expected welfare gain. From Jensen's inequality, for any θ ,

$$u'(c_t) > u' \left(\int c_{t+1} d\psi_\theta \right).$$

This implies that

$$c_t < \int c_{t+1} d\psi_\theta.$$

And applying the iterated rule of conditional expectations, we see that the path of consumption increases monotonically to the steady state:

$$\int c_t d\mu_t \leq \int c_{t+1} d\mu_{t+1}.$$

However this decrease in consumption, ex-ante (at time zero, immediately after implementing the new rule) one-period ahead instantaneous utility increases, due to a redistribution of wealth which provokes a gain in terms of utility that overwhelms the losses. Let write the path of consumption c_t^ε as a function of the history $\{\theta_i\}_{i=0}^t$. Let $h: \Theta \rightarrow \Theta$ be defined in such a way that $h(\theta) = \theta'$ if and only if:

$$\int c_1^\varepsilon(\theta, z) \psi_\theta(dz) = u(c_0 \circ s_0^{-1}(\theta')).$$

Since the divergence of $c_0 \circ s_0^{-1}$ from the identity diverges (in the supremum norm) by an arbitrary small number, the mapping h (which depends on ε) has a unique fixed point $\theta_\varepsilon \in \text{int}\Theta$ for the same reasons pointed out in the first part of the proof. By continuity and monotonicity, this fixed point is unique. Any $\theta \in A_1 \stackrel{\text{def}}{=} [\theta, \theta_\varepsilon]$ satisfies:

$$\int c_1^\varepsilon(\theta, z) \psi_\theta(dz) > u(c_0 \circ s_0^{-1}(\theta)).$$

A similar argument leads to the existence of a family of non empty measurable sets B_t such that, for any $\theta \in B_t$, $(\psi^{(t)} = \psi_\theta \otimes \dots \otimes \psi_\theta)$,

$$\int_{\Theta^t} c_t^\varepsilon(\theta, z)\psi^{(t)}(dz) > u(c_0 \circ s_0^{-1}(\theta)).$$

The set $B \stackrel{\text{def}}{=} \cap B_t$ is non empty because, as the capital is an increasing sequence, so it is the sequence of the wages. Hence, there exists an attainable plan of consumption consisting of holding an amount of money holdings equaling the average $b = 0$, and consuming the labor income. This plan gives a period by period utility strictly greater than the one achieved by the optimal plan under the FR. The proposition is concluded once we show that

$$\lim_{t \rightarrow \infty} \iint u(c_t^\varepsilon) d\psi_\theta d\psi^{(t)} > u(c_0 \circ s_0^{-1}(\theta)). \tag{A.5}$$

From the fundamental theorem of calculus,

$$\int_{\Theta^t} c_t^\varepsilon(\theta, z)\psi^{(t)}(dz) - u(c_0 \circ s_0^{-1}(\theta)) = \int \left(\int_{c_0 \circ s_0^{-1}(\theta)}^{c_t^\varepsilon(\theta, z)} u'(q) dq \right) \psi^{(t)}(dz).$$

Define the measurable mapping from B to its complement $\ell: B \rightarrow B^c$, which is mean-preserving as of period 1:

$$\iint_{B \times \Theta} c_1^\varepsilon d\psi^{(2)} = \iint_{\ell(B) \times \Theta} c_1^\varepsilon d\psi^{(2)}.$$

Since marginal utility is a decreasing function,

$$\int \left(\int_{c_0(\theta)}^{c_1^\varepsilon(\theta, z)} u'(q) dq \right) c_t^\varepsilon \psi_\theta(dz) > \int_{\ell(B)} \left(\int_{c_0(\theta)}^{c_1^\varepsilon(\theta, z)} u'(q) dq \right) \psi_\theta(dz);$$

and

$$\begin{aligned} & \iint_{B \times \Theta} \left(u(c_1^\varepsilon(\theta, \theta')) - u(c_0 \circ s_0^{-1}(\theta)) \right) d\psi^{(2)} \\ & > \iint_{\ell(B) \times \Theta} \left(u(c_1^\varepsilon(\theta, \theta')) - u(c_0 \circ s_0^{-1}(\theta)) \right) d\psi^{(2)}. \end{aligned}$$

Last, we shall argue that agents' long-run utility belonging in the set $B^c \cup \ell(B)^c$ increases, despite reducing their consumption at time 0. The reason is that the previous plan continues to be optimal, since we chose to be close enough to as to make the discrepancy of money holdings with respect to the average negligible. From Theorem 2 in Huggett (1997), the capital stock is increasing. As the initial condition allows households to maintain b up to average, and capital (and wage) grows with σ ,

$$c_t^\varepsilon \geq \underline{\theta} w_t^\varepsilon > \underline{\theta} w_0$$

Since w_0 differs from c_0 by an arbitrary small amount independent of ε , (A.5) must hold. The proposition is concluded.