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Economic Theories in Competition A New Narrative of the Debate on the General Economic Equilibrium Theory in the 1930s

Summary: The paper deals with the debate on the General Economic Equilibrium (GEE) in the 1930s in Vienna and at the London School of Economics and offers an interpretation of it different from that of the traditional narratives. It interprets the debate as a renewed confrontation between the two different classical methodological paths of research in GEE, the Paretian and the Walrasian ones. What emerges from this examination is a picture of different approaches and theories in competition, in particular on the issue of the relationship between theory and the real world. This was the fundamental issue at stake. Herein lies also the interest in those distant controversies for the current debate in economics.

Key words: Hicks, Wald, Von Neumann, General economic equilibrium, Mathematics and economics.

JEL: B23, B41, C02, D50.

General Economic Equilibrium theory (hereafter GEE) is considered the fundamental framework for theoretical discourse in mainstream economics in spite of its well-known limitations - the serious problems connected with uniqueness and stability of equilibrium. In this context, when speaking of GEE, the reference is to its neo-Walrasian version, which is usually considered the rigorous development of Walras's and Pareto's classical theories of GEE. The narrative histories of GEE substantially support this view (see Eliot R. Weintraub 1983; Bruna Ingrao and Giorgio Israel 1990): above all, they emphasise the advance achieved from the analytical point of view in regard to the unsolved formal problems of the predecessors.

This paper assumes a perspective far from the traditional narratives by comparing the epistemologically different research programmes pursued in Vienna and at the London School of Economics (hereafter LSE). Focusing on the three centers where the theoretical issues of the GEE were discussed - LSE, the *Wiener Kreis* and the *Mathematische Kolloquium* - it interprets these discussions as a renewed confrontation between the two different classical methodological paths of research in GEE, the Paretian and the Walrasian ones. It considers the dissent that arose in Vienna and emphasises Wald's and Von Neumann's early foundation of the formalist perspective in economics as the result, not of a cumulative process of knowledge and refinement of mathematical tools, but of an "epistemological break" regarding the nature and method of economics. What emerges from this examination is a picture of different conceptions of what a scientific theory must be, and of theories in competition: another example of that typical phenomenon of "the years of the high theory" (George L. S. Shackle 1967).

1. Mathematical Economics at the Beginning of the Twentieth Century and Pareto's Legacy

In 1900 the fourth edition, the so-called definitive edition, of Walras's *Elements* was published. However, in the first fifteen years of the twentieth century, it was Vilfredo Pareto who made the major theoretical contributions to the developing field of mathematical economics, and Walrasian GEE in particular. His work, from the *Considerazioni sui principi fondamentali dell'economia apura* (Vilfredo Pareto 1892a, b, c; 1893a, b) to the *Manuale di economia politica* (Pareto 1906, 1909), carried the pure economic theory that emerged in the marginalist revolution of the 1870s to its highest stage of development for that time (see Roberto Marchionatti and Enrico Gambino 1997; Marchionatti and Fiorenzo Mornati 2003). At that point, the mathematical system of pure static economics seemed to the contemporary mathematical economists to require only some work for its completion.

As Joseph A. Schumpeter (1954, p. 861) wrote, Pareto's theory was based on Walras's work, and at the same time it deserved to be considered "a new creation". But Pareto's theory was more than a development of Walras's theory on the analytical level. Pareto delineated the methodological bases of an economic science profoundly different from that of Walras (see Marchionatti and Gambino 1997; Marchionatti 1999; Mornati 1999; Pascal Bridel and Mornati 2009). Walras maintained that pure economics is "a physical-mathematical science like mechanics" that uses the "rational method" and not the "experimental method" (Léon Walras 1900, p. 71). Therefore theory is not confirmed by experience but by the structure of theorems and proofs. By contrast, Pareto considered pure economics to be a natural science founded on facts, a science that uses the experimental method peculiar to the natural sciences. Pareto thus initiated a research program in which the definition of assumptions and the empirical verification of theories were fundamental. Pareto's dissent with Walras was clear from his first essays, where he affirmed that the tendency in the mathematical school to subordinate experience to theories was its greatest danger. He criticized Walras' tendency to lead science on a metaphysical path, where reasoning dominates experience, as he wrote in his *Considerazioni*. Experience and observation are the correct methods of reasoning, Pareto maintained. In this context the mathematical method allows for a higher degree of rigour in demonstration, and it enables the treatment of problems far more complicated than those generally solved by ordinary logic. On the other hand, he stressed that economists must use mathematics with caution: the greater rigour of the proof may be only apparent because of the uncertainty of the premises, so that the central theoretical question was the validity of the premises upon which the theorems that yield the conclusions are built (see a letter to Maffeo Pantaleoni, October 3, 1891, in Pareto 1984, pp. 70-71).

Differently from Walras, whose insistence on the rational over the experimental method alienated him from the scientific community, Pareto's work received explicitly positive comments from several scientists and mathematicians in Europe and the United States (see Marchionatti 2004). However, the great Walrasian expectations of the 1870s - the dream of a social astronomy - had greatly diminished in the first fifteen years of the twentieth century. It was generally recognized that the field of mathematical economics is restricted to static equilibrium. In this limited field, the argument ran, the tools of mathematical expression provide a general way to expound the relationship of economic interdependence in a stationary state and are able to specify the conditions and limits of theorems and to prove them rigorously. The discoveries of mathematical economics must not, however, blind economists to the fact that their theories are but static equilibrium theories. Dynamic changes are not taken into account by these theories - in his *Manuale* Pareto introduced the idea of sequences of equilibria, or "successive equilibria" as he called them, but he later became rather skeptical concerning the viability of his project to merge statics and dynamics. Moreover, and this is generally considered to be the main defect of the general equilibrium theory, they are of extreme abstractness. This abstractness makes it difficult to apply the theory's conclusions to the explanation of actual facts. The problem of defining the properties of mathematical functions was considered particularly serious because mathematically "well behaved" functions hardly ever characterize real cases and their indeterminate nature was considered to be the main obstacle to the application of general models to particular cases. It was emphasised (see for example the then well-known book by Wladyslaw M. Zawadski 1914) that there is a great difference between the application of mathematics to economics and its application to mechanics. It was believed that, whilst in mechanics it is always possible to pass from general formulae to actual phenomena, gradually specifying the characteristics of the functions in those formulae, this is not possible in economics. That Pareto was very aware in the last part of his life of these difficulties, and of economic theory's limited ability to explain real economic phenomena, is evidenced by the *Discours du Jubilé* (Pareto 1917) and his increasing interest in sociology.

To summarize, the main issues under discussion in the mathematical economists' community during the early years of the new century were the theory's excessive abstraction, the unreality of its assumptions and models, and the difficulty of explaining real phenomena, rather than its formal aspects. These economists seemed generally not share the preoccupation with the formal establishment of equilibrium that was to dominate mathematical economics later. They were chiefly interested in the problems connected with the relationship between mathematical expression and experimental reality. In their view, the model must be a representation of a real state of the economy, and the main concern of the researcher is not to examine the conditions that must be satisfied in order to be certain that a particular equilibrium exists. This explains their emphasis on the necessity of "not too unrealistic" assumptions.

In the 1920s the theoretical framework of GEE was not modified. The works by the Paretians and Walrasians consisted essentially in presentations of a Walras-Pareto system in textbooks, rather than important creative contributions (see Mario Pomini 2012). In this sense the 1920s mark a decline of the Walras-Pareto perspective. The work of the "grand Paretians" of the 1930s - Hicks first of all - was a sign of the resurrection of the neo-classical general economic equilibrium theory, together with the new mathematical contributions of Wald and Von Neumann in Vienna. However, these contributions were different - profoundly different in the case of the Viennese - from those of the classical period in regard to method and formal analysis.

2. The Neo-Paretian Perspective in London: Hicks's Contribution

2.1 Pareto at the LSE

Paretian economics had a dominant influence on the development of neo-classical microeconomics in the English-speaking world in the 1930s (Marchionatti 2006). A new generation of economists - among them John Hicks, Roy G. D. Allen, Paul Samuelson, Abba Lerner, and Oskar Lange - concentrated on some of the main themes outlined by Vilfredo Pareto's *Manuale*, i.e. the analysis of individual behaviour, market efficiency and welfare economics, using classical differentiable calculus. The story begins at the LSE. In 1929 John Hicks, on Lionel Robbins's request, began to lecture on Walras's and Pareto's general economic equilibrium theories (see John R. Hicks 1973; Omar F. Hamouda 1993). A few years later, he and Allen wrote their famous article "A Reconsideration of the Theory of Value" (Hicks and Roy G. D. Allen 1934a, b), based on the use of the concept of Pareto's indifference curves, where they introduced the concepts of income and substitution effects, as well as the principle of marginal substitution - the article was essentially a rediscovery of Eugen Slutsky (1915) (see Allen 1936). One year before, in 1933, some LSE graduate students had founded the *Review of Economic Studies*, a journal which contributed greatly to acquaintance with Pareto in the UK and published many articles on Paretian themes (see Marchionatti 2006). Pareto's work on pure theory "important as it is, and influential as it has been" (Hicks 1939, p. 3) was considered "a beginning" to be developed along the new lines laid down by Slutsky, Allen and Hicks, implicitly reintroducing the successive approximation approach which the late Pareto *de facto* abandoned. Indeed, the achievements in microeconomic theory at LSE between 1934 and 1939 (the year of Hicks's *Value and Capital*) were noteworthy. Hicks's book was the most ambitious outcome of this neo-Paretian approach.

2.2 Hicks's Value and Capital

Value and Capital is a work on theoretical economics based on the idea that the problem of economic theory at that time was the construction of "a technique for studying the interrelations of markets" (Hicks 1939, p. 2) assuming as a starting point Walras's and Pareto's work (and Wicksell's for his consideration of the dynamic problem of capital): "our work", Hicks writes in the introduction: "is bound to be in their (Walras's and Pareto's) tradition, and to be a continuation of theirs" (ibid.). It was considered at that time a standard work, a classic in the field.

The book begins by presenting the new theory of subjective value based on the concept of ordinal utility and then applies the result to rework the GEE static analysis of Walras and Pareto. In the first two sections Hicks grounds the static theory of general economic equilibrium of perfect competition on a theory of demand which is a development of Pareto's theory of choice and a theory of production which is an adaptation of demand theory to the study of firm behaviour. Hicks focuses on the static theory of exchange because, according to himself, in both the static equilibrium and the dynamic equilibrium of production "almost exactly the same questions" of the exchange theorycome up: "this is why the theory of exchange is an essential part of the study of the economic system in general" (Hicks 1939, p. 77).

Hicks starts from the equilibrium of the consumer in conditions of perfect competition in its "classic" version. Then he introduces the condition for stability of the equilibrium: $du = 0$ and $d^2u < 0$, where u is the utility function, and emphasizes that this condition do not depend on the existence of a particular utility function: it can be deduced from any arbitrary utility function $\varphi(u)$ provided that $\varphi'(u) > 0$. After examining the effect on demand of an increase in income and the effect of a change in price with constant income, Hicks defines the equation, originally formulated byS-lutsky, which he regarded as "the Fundamental Equation of Value Theory" (Hicks 1939, p. 309), which divides the effect of any price change on demand into two effects: the income effect and the substitution effect. Hicks then considers the existence of the equilibrium of exchange in a world where N individuals exist, "bringing to the market various quantities of n goods, and exchanging them under conditions of perfect competition" (Hicks 1939, p. 314). After showing that there are $n - 1$ equations to determine the $n - 1$ prices, Hicks maintains that the equilibrium exists. According to Hicks, having shown, *à la* Walras, "the mechanism of the interrelation of markets … was a great achievement", but of a certain sterility, because it is not clear how this mechanism works, i.e. the laws of change are not discussed: the mechanism does not explain, Hicks writes, "what would happen if tastes and resources changed" (Hicks 1939, p. 61). But, he believes, "with the technique now at our disposal (that is, the new theory of demand), we can make a similar investigation for the general case" (ibid.) and counter most of the accusation of sterility brought against the GEE. The laws of change of the price system - writes Hicks - "like the laws of individual demand, have to be derived from stability conditions" (Hicks 1939, p. 62). According to Hicks, the GEE is stable if a slight movement away from the equilibrium position should set up forces tending to restore equilibrium. He calls the difference between the demandand supply at any price "the excess demand" for a good. He then reformulates the equilibrium condition of a market as the condition that the excess demand should be zero, and the condition of stability of equilibrium as the condition that the excess demand should increase when the price falls and decreases when the price increases. The stability conditions enable us to study the variation of prices following a change in consumer preferences towards a certain good. The analysis concludes that "a moderate degree of substitutability" among the goods is sufficient to prevent "the only possible ultimate source of instability", the asymmetry in the income effects, thus assuring stability of the equilibrium in conditions of perfect competition.

This reworking is followed by the "foundations of dynamic economics", where Hicks discusses the problem of intertemporal equilibrium and presents his model of temporary equilibrium following the path laid down by Friedrich A. von

Hayek (1933). Finally, it deals with the workings of a dynamic system in order to create a theory of the economic process over time. The dynamic parts of the book are certainly a truly innovative contribution: Hicks proposed a temporary equilibrium model where the path of the economy follows a sequence of temporary equilibria - the idea proposed by Pareto in his *Manuale* - and introduced the idea of "pure future economy" which inspired the axiomatic model of intertemporal general equilibrium developed by Arrow and Debreu. These dynamic parts are, however, not organically connected with the previous parts, due primarily to Hicks's subjective requirement of describing the dynamics of real economies as well as the statics. In fact, in general, Hicks' aim was to respond to the criticism of static and highly abstract representation of the economy, and the accusation of sterility brought against the Lausanne theory by Marshallians in England. Therefore, the aim of his theoretical project was to find a way out of the *impasse* into which the GEE theory had become trapped in its classical era, while at the same time maintaining continuity with that work. In this project, the reworking of that theory, consistently with the LSE perspective, was fundamental because - and this should be stressed - it was considered to represent the foundation of the economic laws operating in reality.

2.3 Value and Capital: "A Revolutionary Book" or "an Old-Fashioned Book"?

At the time of its publication *Value and Capital* was widely considered, above all in the English-speaking world, a work reconciling the theory of equilibrium and the theory of real dynamics, and in any case a revolutionary book in the Paretian perspective (for a synthesis of the reviews see Warren Young 1991, 2008). It certainly inspired the work of English-speaking economists, in particular Paul Samuelson, who in the *Preface* to the *Foundations of Economic Analysis* recognised the similarity of points of view with *Value and Capital* and, fifty years later, confirmed that Hicks was one of the few economists of the 1930s who received his "attention" (Paul A. Samuelson 1998, p. 1381). Samuelson was also critical of *Value and Capital*. In particular, he showed that the Hicksian stability conditions are not, in general, necessary nor sufficient in order to satisfy the stability of equilibrium in a dynamic system (Samuelson 1941, pp. 108-112; Samuelson 1948, pp. 269-274). From his point of view, Hicks later wrote that Samuelson, Arrow et al. recognized in *Value and Capital* the starting point of their work on GEE then accomplished "with far more skill in mathematics" than himself. On the other hand, Hicks judged those "great" results extraneous to his way of thinking, which did not adhere to making theory for its own sake nor econometrics (Hicks 1979, pp. 201-202). Roy Allen, who was at that time in constant intellectual dialogue with both Hicks and Samuelson, wrote (Allen 1949, p. 112) that the two authors "have now come together on essentials" so that the "future development … will flow from an agreed combination of the two expositions". This judgement became the general opinion: Kenneth J. Arrow (1974, pp. 255-260) speaks of a "Hicks-Samuelson Model of General Equilibrium" whose "primary interest … was rather in the laws of working of the general equilibrium system … than in the questions of existence and the like".

On the other hand, a highly critical examination of the book was conducted in 1941 by Oskar Morgenstern, at that time professor at Princeton. Morgenstern's criticism focused on the formal problems of the book: he accused Hicks of lacking rigour and of being outdated. The main point raised by Oskar Morgenstern (1941, p. 368) was "a fundamental issue … which has loomed large in the writings of mathematical economists ever since Walras. It is the question as to whether the determinateness of any given economic system … is assured when the number of unknowns involved equals the number of equations that can be set up". Establishing such a system, Morgenstern maintained, would be a major achievement, and the main issue was whether or not the system of equations embodied all the assumptions that the economist has to include. But, he observed, relatively scant attention was paid to this question in the mathematical treatment of economic theory: the mathematical economists of the classic era (from Walras to Fisher, Cassel and Pareto) had failed, Morgenstern maintained, "even to see the task that was before them", and, he declared, "Professor Hicks has to be added to this list" (Morgenstern 1941, p. 369). Only very recently, Morgenstern noted, had "an important step forwards … been made, due exclusively to mathematicians and not to economists" (Morgenstern 1941, p. 369). He was referring to the work of John von Neumann and Abraham Wald in Vienna, which Hicks had ignored. Needless to say, his criticism reflected the influence of Von Neumann (Robert Leonard 2010). Morgenstern (1976) recounts Von Neumann's irreverent opinion of the productions of mathematical economics at the end of the 1930s. He considered their mathematics "primitive" and economics "simply still a million miles away from the state in which an advanced science is, such as physics": it seems that this judgement referred particularly to Hicks (see Ingrao and Israel 1990, p. 197 and note 60 p. 410). Substantially the same judgement was passed on Samuelson: Leonard (2010, p. 244) writes that Von Neumann felt a private disdain for the "primitivity" of the Samuelsonian mathematical economics of *Foundations*.

3. Neo-Paretian and Neo-Walrasian Perspectives in Vienna

3.1 Premise: The Viennese Debate

Between the two world wars economic discussion was lively in Vienna, not only in the Austrian School's seminars run by Hans Mayer and Ludwig von Mises, but also in the two philosophical and scientific circles: the *Wiener Kreis*, created at the beginning of the 1920s by the physicist and philosopher Moritz Schlick; and the *Mathematische Kolloquium* founded in 1928 and conducted by the mathematician Karl Menger, and attended mainly by logicians and mathematicians (on Menger's role in interwar Vienna, see Leonard 1998). Although economic discussion was only a small part of the debate in these two circles, it was a stage of crucial importance in the relationship among economic theory, philosophy and mathematics in the twentieth century. Traditionally, the literature on the history of economic ideas does not distinguish between the approaches to economics of the two circles, associating them as expressions of the new path of scientific discourse. More importantly, there has been general consensus that there was some sort of continuity and intellectual interchange between the *Kreis* and the *Kolloquium* (Donald Gilles 1981; Weintraub 1983, 1985, 2002; Lionello F. Punzo 1989, 1991; Ingrao and Israel 1990; Louise A. Golland 1996; Isao Mutoh 2003). In fact, there is evidence that the two Viennese circles came to express different conceptions of the scientific discourse (Giandomenica Becchio and Marchionatti 2005). The *Kreis*'s members adopted Bertrand Russell's logicism and the experimental approach to reality taken by physics. By contrast, the *Kolloquium*'s members adhered to David Hilbert's mathematical formalism and adopted a deductive and highly formalised method. The mathematicians of the *Kolloquium* rejected the *Kreis*'s "physicalism" and tended to downplay the importance of the verificationist paradigm. These different philosophical conceptions were reflected in different conceptions of economic theory: a conception of economics as empirical science supported by the *Wiener Kreis*; and a conception of economics as a mathematical science supported by the *Mathematische Kolloquium*.

3.2 Economics as an Empirical Science: The Neo-Paretian Perspective in the Wiener Kreis

Logical empiricism was founded by the *Wiener Kreis*. This new empiricism shared with its predecessor the assumption that knowledge starts from the observation of empirical data. However, the new empiricists added that the statements made by empirical sciences are connected and ordered by a new logical analysis (Otto Neurath 1973): the discovery of new statements (laws or theorems) is the aim of all scientific research. But they rejected all logically or empirically unverifiable statements as meaningless. This new view of science was the philosophical foundation of the *Kreis*'s scientific vision as described in the *Manifesto* written by Otto Neurath together with Hans Hahn and Rudolf Carnap (Rudolf Carnap, Hans Hahn, and Neurath 1929). This vision was developed into the subsequent concept of unified science (*Einheitswissenshaft*), whose language was so-called physicalism - i.e. the application of the language of physics to the other sciences.

In the *Manifesto*, economics was placed among the five branches of science that "must conduct an epistemological examination of its foundations, a logical analysis of its concepts" (Neurath 1973, p. 315) in order to purge it of metaphysical residuals. To be noted is that this was also a crucial point in Pareto's research program. In fact, Pareto's thought had an influential - but neglected in the literature - role in the *Kreis* (see Marchionatti 2006). It was known through Neurath's writings: Neurath (2004), already in his economic writings before the war, referred in particular to the *Manuel* and an important Paretian text published in the *Enzyklopadie der mathematischen Wissenschaften* (Pareto 1902), which influenced his approach to economics. But what should be emphasized is not only that there was explicit understanding of Pareto's work in the *Kreis*, but also that it anticipated the *Kreis*'s theoretical conception of the scientific nature of economics. Both the members of the *Kreis* and Pareto pursued the goal of creating a unified science. Both stressed the importance of the verificationist paradigm. The *Kreis*'s relationship with Pareto is not surprising if we consider Pareto's attention to the development of science and his increasing agreement with the ideas put forward by Poincaré - one of the authors at the center of the initial reflections of the *Kreis* - at the beginning of the twentieth century (Marchionatti and Gambino 1997). Francis Y. Edgeworth (1915) first pointed out the similarity between Pareto's and Poincaré's positions on the ordinalist conception of the utility function, a crucial point in Pareto's methodological and theoretical revolution and in his separation from Walras (see Marchionatti and Gambino 1997). For these reasons we can maintain that the disagreement between the *Wiener Kreis* and the *Mathematische Kolloquium* over the economic discourse is analogous to the old classical disagreement between Walras and Pareto over the nature and method of political economy referred to in Section 1 above. This is what makes Pareto, from the epistemological point of view, a precursor of the Viennese neo-positivism in economics.

Another important connection between Pareto and the *Kreis* may be emphasized. In a paper presented at a conference held in Paris in 1935, and whose purpose was to submit the program of the *International Encyclopaedia of Unified Science* (the development in the mid-1930s of the *1929 Manifesto*) to the scientific community, the French economist, engineer and statistician Robert Gibrat (1936) explicitly associated the *Kreis*'s program with the recently-born econometric movement. Econometrics emerged as one of the "modern models" that conceived economic theory as the field of application of exact logic. It adopted the methods of natural science that would assure the clarity and rigor necessary for theory and empirical research in economics, and the epistemological paradigm of verificationism which originated in the *Kreis* circle. These modern models were explicitly conceived as applied developments of Walras's and Pareto's mathematical economics, as Jan Tinbergen (1949) pointed out. The early econometricians - among them Paretian mathematical economists such as Luigi Amoroso, François Divisia, Wladyslaw M. Zawadski, Henry Schultz, Edwin B. Wilson, all of them members of the first Advisory Board of *Econometrica*, the official journal of the Econometric Society launched in 1933 - continued to give fundamental importance to Pareto's main methodological problems - i.e. the issues of the excessive abstraction of pure economics, the realism of the key assumptions and models, and the relationship between the mathematical formulation of the models and experimental reality. By contrast, a totally different epistemological approach was adopted by "the economists" of the *Mathematische Kolloquium*.

3.3 Economics as a Mathematical Science: The Neo-Walrasian Perspective in the Mathematische Kolloquium

3.3.1 Premise

Karl Menger had been a regular participant in the *Kreis*, but he soon had reservations about developments in the *Kreis*'s original empiricism (Karl Menger 1994; Seymour Kass 1996). The publication of the *Manifesto* left him skeptical, to the point that he called it "rather superficial" (Menger 1974, p. 114) - and later criticised the programme of the "unity of science". He consequently founded the *Mathematische Kolloquium* in 1928. In the *Kolloquium*, studies on the contemporary development of geometry and logics, as well as "studies concerning the new applications of exact sciences to problems of sociological character", were carried out (Menger 1935, p. 327). In a note to this article he added: "for example on the existence and uniqueness of solutions for the production equations in mathematical economics".

"The real link between Walras … and the nascent developments" (Weintraub 1983, p. 6) was the banker and economist Karl Schlesinger. He addressed the issue of the existence of economically meaningful (positive) solutions in the Walrasian model in a short paper published in 1935 in the *Ergbenisse eines Mathematischen Kolloquium* (the proceedings of the *Kolloquium*) and previously presented, at Menger's invitation, to the *Kolloquium*. The model considered by Schlesinger was the so-called Walras-Cassel system based on Gustav Cassel's simplified reformulation of the Walrasian general economic equilibrium (Gustav Cassel 1899, 1918): Schlesinger and Menger became familiar with the Walras-Cassel system on attending Hans Mayer's seminar in Vienna (Mutoh 2003). Kurt Schlesinger (1935), like Hans Neisser (1932), Frederik Zeuthen (1932) and Heinrich von Stackelberg (1933) before him, emphasized that the equality between the number of equations and the number of unknowns does not necessarily mean that the system possesses positive solutions in prices. He reformulated Cassel's system in terms of inequalities, but without going on to its mathematical solution. Schlesinger's paper opened the way for Abraham Wald's work (on Wald and his formative years see Morgenstern 1951 and Menger 1952). In a series of trail-blazing papers (Abraham Wald 1935, 1936a, b) Wald demonstrated the existence of an equilibrium for the Walras-Cassel system.

From a methodological point of view, Wald's work took as its premise Menger's conception of meta-economics - a meta-theory corresponds to the logical relations between the statements of a theory (Menger 1936; see also Becchio 2009). Menger (1994) claimed that "from the point of view of methodology", his 1936 paper was "the first instance in economics of a *clear separation* between the question of logical interrelations among various propositions and the question of empirical validity" (Menger 1994, p. 300). According to him, it was the key point needed to transform economics into a science. This "clear separation" between the question of logic and the question of empirical validity, which Schumpeter (1954, p. 1037) described as "a shining example of the general tendency towards increased rigor that is an important characteristic of the economics of our own period", is at the basis of Wald's work and of the programme for the new mathematization of Walrasian general economic equilibrium theory. According to the historical reconstructions (Weintraub 1983; Arrow 1989), Menger showed Wald's paper to Von Neumann and invited him to publish in the *Ergebnisse* his 1932 paper on general economic equilibrium dynamics read to the *Princeton Mathematical Society*. Von Neumann's paper (John von Neumann 1937), translated in English in 1945 as "A Model of General Economic Equilibrium" (Von Neumann 1945) was a more advanced mathematical formalization, from the technical point of view, of the problem of the existence of an equilibrium, and it freed the model from any bond with the real world, which still existed in Wald's methodological premise (see Israel and Ana Millàn-Gasca 2009; Nicola Giocoli 2010; Sandye Gloria-Palermo 2010).

3.3.2 Abraham Wald's Contribution: Between Tradition and Innovation

In his 1936 expository article, Abraham Wald (Wald 1936b, p. 368) started by maintaining that "mathematical economics" is "a new method", and "an indispensable tool for many subtle investigations of various areas of economic phenomena" (ibid.).

Unfortunately, he added, "sins have been committed in mathematical economics": unawareness of the assumptions and their implications, and of their conditions of validity. These sins are not imputable to mathematical method itself, Wald claimed, for "they have their origin in inappropriate, even erroneous, applications of mathematics" (ibid.). He thought that for a "fruitful application of mathematics in economics" it was essential that all the assumptions "be enumerated completely and precisely" (ibid.). These recommendations had already been strongly emphasized by Menger in his 1936 paper. According to Wald, "if these directions are strictly adhered to", then "the only objection which can be raised against a theory is that it includes assumptions which are foreign to the real world and that, as a result, the theory lacks applicability" (Wald 1936b, p. 369) - the key issue according to Pareto and the Paretian mathematical economists. Wald recognized that "in many areas of mathematical economics very substantial abstractions are being used, so that one can hardly speak of a good approximation to reality" (ibid.), but he defended the adoption of "far-reaching abstractions" using some (weak) arguments that had already been used by some mathematical economists of past generations - i.e. that "mathematical economics is a very young science" (ibid.) and "economic phenomena are of such a complicated, involved nature that far reaching abstractions must be used at the start merely to be able to survey the problem" (ibid.). These problems required adopting the method of successive approximations - "more realistic assumptions must be carried out step by step". Wald optimistically concluded that if these "directions are strictly adhered to", then "it will always be known precisely just where the assumptions are still so simplified and unrealistic that they must be replaced with better ones, so that ultimately theories will be derived that are well applicable to the real world" (ibid.).

Wald was the first to deal with the mathematical questions of existence and uniqueness in a systematic way. He started by criticizing the assumptions made by the old mathematical economics on the equality of the number of equations and unknowns, recalling that: "the equality of the number of equations and unknowns does not prove that a solution exists, much less the uniqueness of a solution" (Wald 1936b, pp. 369-370). The assumption that this equality may represent a sufficient condition for the solution of the system of equations is inadequate in the economic field because solutions have economic meaning only if they are non-negative in the prices of goods and services. Therefore Wald investigated the conditions of non-negativity. He adopted the simplified version of Walras's equations proposed by Cassel with the modifications introduced by Schlesinger (1935). Cassel's model assumed that the demand function depends on the prices of all commodities; by contrast, the Schlesinger-Wald model followed the Austrian school's approach and inverted the relationship, that is, used inverse demand functions in which prices are determined by the quantity of demand (see Punzo 1989). The Cassel system is written as follows:

$$\begin{gathered} r_i = \Sigma a_{ij} s_i \; (i = 1, \ldots, m; j = 1, \ldots, n) \\ \sigma_j = \Sigma a_{ji} \rho_j \; (i = 1, \ldots, m; j = 1, \ldots, n) \\ \sigma_j = f_j(s_1, s_2, \ldots, s_n) \; (j = 1, \ldots, n) \end{gathered} \tag{1}$$

where:

$r_1, r_2, \ldots, r_m$ are the available quantities of the m productive services $R_1, R_2, \ldots, R_m$
$s_1, s_2, \ldots, s_m$ are the quantities produced of the n goods $S_1, S_2, \ldots, S_n$
$\sigma_1, \sigma_2, \ldots, \sigma_n$ are the prices of n goods
$\rho_1, \rho_2, \ldots, \rho_m$ are the prices of the m productive services
a_{ij} are the technical coefficients considered constant $(i = 1, \ldots, m; j = 1, \ldots, n)$.

Wald considers as production factors all the factors, both scarce (those considered by Walras and Cassel) and not scarce, or free. This implies the transformation of the equations into inequalities. Hence we have:

$$r_i \geq \sum a_{ij} s_j \quad (i = 1, \ldots, m; j = 1, \ldots, n) \tag{2}$$

or

$$r_i = \sum a_{ij} s_j + u_i \quad (i = 1, \ldots, m; j = 1, \ldots, n) \tag{3}$$

where $\forall i$, $u_i \geq 0$. If $u_i > 0$ then r_i is a free good and $\rho_i = 0$. This means adding m equations:

$$u_i \rho_i = 0 \quad (i = 1, \ldots, m). \tag{4}$$

The problem to be solved is demonstrating the existence of a economically meaningful solution of the system of $2m + 2n$ equations in $2m + 2n$ unknowns, i.e. $u_i, \rho_i, s_j, \sigma_j, (i = 1, \ldots, m; j = 1, \ldots, n)$. The new system of equations is:

$$\begin{aligned} r_i &= \sum a_{ij} s_i + u_i \quad (i = 1, \ldots, m; j = 1, \ldots, n) \\ u_i \rho_i &= 0 \quad (i = 1, \ldots, m) \\ \sigma_j &= \sum a_{ji} \rho_j \quad (i = 1, \ldots, m; j = 1, \ldots, n) \\ \sigma_j &= f_j(s_1, s_2, \ldots, s_n) \quad (j = 1, \ldots, n) \end{aligned} \tag{5}$$

where r_i and a_{ji} are given and f_j are known functions. Wald showed the existence of economically meaningful solutions under a set of (limitedly realistic, according to himself) hypotheses.

3.3.3 Beyond Tradition: John von Neumann's Contribution

On the intellectual origin of the model. Arrow (1989) thought that there was a Walrasian influence in Von Neumann's model, and this has been the prevailing interpretation in the literature. However, subsequent historical research has shown that the intellectual origins of the model are not so simple. They seem to derive not only from the Viennese discussion on the Walras-Cassel model but also from the Berlin debate in Ladislaus von Bortkievicz's circle (Marchionatti and Raffaella Fiorini 2000). According to some scholars (Waldemar Wittmann 1967; Heinz D. Kurz and Neri Salvadori 1993; Leonard 1995), Von Neumann may have been influenced by the price model in a planned economy formulated by Robert Remak (1929). Remak was a young mathematician, student of the mathematicians Georg Frobenius and H. A. Schwarz, who had an intellectual relationship with Bortkievicz (Wittmann 1967). He was *privat-dozent* at the University of Berlin from 1929 to 1933, more or less the same period in which Von Neumann was there (Stanislaw Ulam 1958). The simultaneous presence in Berlin of Von Neumann and Remak gave rise to the conjecture, advanced by Wittmann (1967) and revived by Kurz and Salvadori (1993) and also accepted by Leonard (1995), that in preparing his model, the young Von Neumann had in mind the model that his older colleague had presented at a Berlin Mathematical Society seminar.

In 1929, following Bortkievicz's suggestions (Remak 1933), Remak carried out a study on the determination of rational prices for a centrally planned closed economy. His model represented the economy as a classical circular process of production. He considered a closed economy without wages and profits in which the quantities of the various commodities produced and consumed are known. The problem for such a system is determination of a set of prices which would "provide the basis for a financially viable economy" (Remak 1929, p. 271). Under the assumptions that the production process is circular, the total quantities of each product and the productive technology are given, the period of production is the year, and the system is in a stationary state, the problem to solve is the following: given the technical coefficients of production, $a_{ij} \geq 0$ ($i, j = 1, \ldots, n$), i.e. the quantity of each commodity that the industry i furnishes to the industry j to produce a unit of the commodity j (which could be positive or zero in the case in which the industry i does not supply anything to the industry j), to determine the prices of the commodities y_i ($i = 1, \ldots, n$) so that each industry's income from the supplied commodities is equal to its expenditure on the received goods. This system is called a "superimposed price system" or "rational price system". Remak demonstrates the existence of an economically relevant solution, i.e. where prices are ≥ 0, unique up to a factor of proportionality.

This "classical" representation is the one adopted by Von Neumann in his 1937 paper. The objective affinity between Von Neumann's view of the economy and the classical economists' approach was emphasized for the first time by David G. Champernowne (1945) in his paper accompanying the English publication of Von Neumann's paper and closely discussed by the Italian economist Claudio Napoleoni (1965).

Von Neumann's model. The model assumes a linear technology of a set of-processes of production and goods. It is a "closed" circular model because there is no distinction between resources and final uses - "goods are produced not only from natural factors of production, but in the first place from each other" (Von Neumann 1945, p. 1). Calling a_{ij} and b_{ij} the units of G_j ($j = 1, \ldots, n$) respectively consumed and produced in P_i ($i = 1, \ldots, m$) with $a_{ij} \geq 0$ and $b_{ij} \geq 0$, the process may be described as follows:

$$P_i : \sum_{j=1}^{n} a_{ij} G_j \rightarrow \sum_{j=1}^{n} b_{ij} G_{j_{j_i}} . \tag{6}$$

Each process is considered to be "of unit time duration" ("processes of longer duration to be broken down into single processes of unit duration") (Von Neumann 1945, p. 2). Processes are used with a certain intensity. Von Neumann defines equilibrium as the state "where the whole economy expands without change of structure" (ibid.), i.e. where the ratios of the intensities are "unchanged" (ibid.). They must be multiplied by a common factor α per unit of time, α being the ratio between production in a period of time and production in the preceding period - i.e. "the coefficient of expansion of the whole economy" (ibid). Given the quantities a_{ij} and b_{ij}, it is necessary to determine: (i) the intensities of the processes; (ii) the coefficient of expansion of the whole economy; (iii) the prices of goods; (iv) the interest factor. This gives rise to a system of inequalities. In order to solve this system, Von Neumann made use of mathematical methods much newer than Wald's, and able to improve greatly on his proofs. In fact, Von Neumann's method of analysis did not use differential techniques but instead employed topological techniques for the first time in economics. The demonstrative technique transformed the problem of determining an equilibrium into a *minimax* problem: that is, the conditions of existence of an equilibrium are equivalent to the condition necessary for a *minimax* solution (a saddle point). The solution of the system of equations is possible, Von Neumann wrote, "only by means of a generalization of Brouwer's Fixed-Point Theorem - i.e. by the use of very fundamental topological facts" (Von Neumann 1945, p. 1). This connects the solution of systems of linear inequalities to the *minimax* solution of a two-person zero-sum game of a previous 1928 article where Von Neumann laid the mathematical bases of game theory and proved the first *minimax* theorem. In a note to the paper, Von Neumann emphasized the connection: "the question whether our problem has a solution is oddly connected with that of a problem occurring in the Theory of Games dealt with elsewhere (Math. Annalen, 1928)" (Von Neumann 1945, p. 5). Whereas Von Neumann proved the existence of a saddle point for a certain function in the 1928 paper, in the 1937 paper he proved a "fixed point lemma" that generalizes Brouwer's theorem, from which the existence of a saddle point for the equilibrium function follows (see Tinne H. Kjeldsen 2001). The use of a fixed-point theorem in the proof of existence of equilibrium became a standard tool in general equilibrium analysis, one of the technical cornerstones of the modern approach. We know that also Wald presented a fixed-point proof of a general economic equilibrium model to Menger's *Kolloquium* in the fall of 1935 but, being scheduled for the ninth issues of the series, it could not be printed in the *Ergebnisse* because of the demise of the *Kolloquium* after the annexation of Austria to Nazi Germany in 1938. That proof never appeared in print, nor was it circulated by Wald. Recent new evidence from the Karl Menger papers, however, proves its existence. The story of this discovery is narrated in Till Düppe and Weintraub (2015).

Axiomatic approach in a totally coherent way. As many scholars have emphasized (for example Ingrao and Israel 1990; Punzo 1991), the axiomatic approach in economics - on the difference between axiomatics and formalism see Weintraub 1998, 2002; Philippe Mongin 2003; Elettra Agliardi 2004 - is applied in Von Neumann's paper in a totally coherent way, in the sense that the concern for the economic interpretation of the model - still existing in Wald - disappears. This theoretical attitude derived, firstly, from the fact that, as Champernowne noted in his "commentary note" to the English translation of Von Neumann's paper (1945), Von Neumann dealt with the economic question "as a mathematician". In this way he obtained a mathematical solution of a "highly generalised problem in theoretical economics" characterized by the elegance of its solution, logical completeness, concision and rigor, but he adopted "extremely artificial assumptions" (Champernowne 1945, p. 10), or "idealisations" as Von Neumann termed them. Secondly, Von Neumann's attitude derived from the fact that he dealt with theoretical economic problems like a *formalist mathematician* - i.e. he conceived the model as a formal structure whose legitimacy and cogency depend on its internal consistency.

Von Neumann's axiomatic program set out in the 1937 paper, and Menger's reflections, provide theoretical justification for the weak link between theory and the real world in Walras - a well-known problematic issue emphasised by Poincarè in his correspondence with Walras himself. Von Neumann's 1937 program freed the Walrasian one from the need for the realism of hypotheses and their verification. He abandoned the idea of mathematics as logic and rigor, *together with* the strong emphasis on facts and applications that predominated among mathematical economists of the Paretian school of the preceding period, and among contemporary mathematical economists and econometricians. In this sense the debate on the Walras-Cassel model in the *Mathematische Kolloquium* was the beginning of the neo-Walrasian theory of the 1950s. It laid the bases for the radical extension of formalism in economics definitively affirmed with the Bourbakist Gérard Debreu (1959): "allegiance to rigor dictates the axiomatic form of the analysis where the theory, in the strict sense, is logically entirely disconnected from its interpretations", states Debreu, adding that such a dichotomy between the theory in the strict sense and its interpretation "reveals all the assumptions and the logical structure of the analysis" and "makes possible immediate extensions of that analysis without modification of the theory by simple reinterpretations of the concepts" (Debreu 1959, p. x).

4. Conclusions

Narrative histories of the GEE in the 1930s have some major shortcomings. They focus essentially on the *Mathematische Kolloquium*'s contributions in Vienna, considering them to constitute a revolutionary new step in economic theory and the rigorous development of Walras's and Pareto's classical theories of general equilibrium. They omit the *Wiener Kreis*'s reflections on the nature and method of economics. They underestimate, considering it old-fashioned compared with the works of Viennese mathematicians, Hicks's contribution at LSE.

By focusing on the theoretical production and the methodological discussion in Vienna and at the LSE, this paper has outlined a new interpretation of the debate on the GEE in the 1930s in terms of "theories in competition" on the issue of the relationship between theory and the real world: this, in fact, was the fundamental issue at stake. As regards Hicks, his theoretical project had the explicit ambition of bridging the gap between statics and dynamics in the GEE model, thereby resolving the sterility of the classical model and the impasse in which it was trapped, but still remaining within the Paretian analytical and methodological framework. As regards the *Wiener Kreis*'s reflection on the relation between theoretical model and the real world, it represents, from the epistemological and methodological point of view, a "modernisation" of Pareto's thought. As regards the *Mathematische Kolloquium*'s contribution by Menger, Wald and Von Neumann, they took the abstractness of the

Walrasian model to its extremes in their economic thought. In this theoretical "competition", two different conceptions of rigor emerge: the rigor of (experimental) method recommended by Paretians, and the mathematical (formal) rigor recommended by the neo-Walrasians Menger, Wald and Von Neumann. That "competition" was not between innovators and conservative, old-fashioned positions, but rather between different conceptions of economics and methodological approaches.

References

Agliardi, Elettra. 2004. "À propos de... 'l'axiomatisation et les théories économiques' de Philippe Mongin." *Revue économique*, 55(1): 123-142.

Allen, Roy G. D. 1936. "Professor Slutsky's Theory of Consumers' Choice." *Review of Economic Studies*, 3(2): 120-129.

Allen, Roy G. D. 1949. "The Mathematical Foundations of Economic Theory." *Quarterly Journal of Economics*, 63(1): 111-127.

Arrow, Kenneth J. 1974. "General Economic Equilibrium: Purpose, Analytic Techniques, Collective Choice." *American Economic Review*, 64(3): 253-272.

Arrow, Kenneth J. 1989. "Von Neumann and the Existence Theorem for General Equilibrium." In *John von Neumann and Modern Economics*, ed. Mohammed Dore, Sukhamoy Chakravarty, and Richard Goodwin, 15-28. Oxford: Clarendon Press.

Becchio, Giandomenica, and Roberto Marchionatti. 2005. "Teoria economica, filosofia e matematica. La riflessione a Vienna nel Wiener Kries e nel mathematische kolloquium, 1922-38." Centro di studi sulla storia e i metodi dell'economia politica "Claudio Napoleoni" Working Paper 08/2005.

Becchio, Giandomenica. 2009. "A Historical Note of the Original Meaning of Metaeconomics." *Intellectual Economics*, 1(5): 7-11.

Bridel, Pascal, and Fiorenzo Mornati. 2009. "De l'équilibre général comme 'branche de la métaphysique', ou de l'opinion de Pareto sur le projectwalrasien." *Revue économique*, 60(4): 869-890.

Carnap, Rudolf, Hans Hahn, and Otto Neurath. 1929. *Wissenschaftliche Weltauffassung: der Wiener Kreis*. Wien: Arthur Wolf Verlag.

Cassel, Gustav. 1899. "Grundriss einer elementaren Preislehre." *Zeitschrift für die gesamte Staatswissenschaft*, 55(3): 395-458.

Cassel, Gustav. 1918. *Theoretische Sozialökonomie*. Leipzig: C. F. Winter.

Champernowne, David G. 1945. "A Note on J. v. Neumann's Article on 'A Model of Economic Equilibrium'." *Review of Economic Studies*, 13(1): 10-18.

Debreu, Gérard. 1959. *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*. New Haven: Yale University Press.

Düppe, Till, and Eliot R.Weintraub. 2015. "Losing Equilibrium: On the Existence of Abraham Wald's Fixed-Point Proof of 1935." Center for the History of Political Economy Working Paper 2015-04.

Edgeworth, Francis Y. 1915. "Recent Contributions to Mathematical Economics." *Economic Journal*, 25(97): 36-63.

Gibrat, Robert. 1936. "La science économique. Methodes et philosophie." In *Actes du congrès international de philosophie scientifique*, ed. Alfred Tarski et al., 22-31. Paris: Hermann & Cie.

Gillies, Donald. 1981. "Karl Menger as a Philosopher." *British Journal for the Philosophy of Science*, 32(2): 183-196.

Giocoli, Nicola. 2010. "John von Neumann's Panmathematical View." *History of Economic Ideas*, 18(1): 209-217.

Gloria-Palermo, Sandye. 2010. "Introducing Formalism in Economics: The Growth Model of John von Neumann." *Panoeconomicus*, 57(2): 153-172.

Golland, Louise A. 1996. "Formalism in Economics." *Journal of the History of Economic Thought*, 18(1): 1-12.

Hamouda, Omar F. 1993. *John R. Hicks. The Economist's Economist*. Oxford: Blackwell Publishing.

Hicks, John R., and Roy G. D. Allen. 1934a. "A Reconsideration of the Theory of Value. Part I." *Economica*, 1(1): 52-76.

Hicks, John R., and Roy G. D. Allen. 1934b. "A Reconsideration of the Theory of Value. Part II. A Mathematical Theory of Individual Demand Functions." *Economica*, 1(2): 196-219.

Hicks, John R. 1939. *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory*. Oxford: Clarendon Press.

Hicks, John R. 1973. "Recollections and Documents." *Economica*, 40(157): 2-11.

Hicks, John R. 1979. "The Formation of an Economist." *Banca nazionale del Lavoro quarterly review*, 32(130): 195-204.

Ingrao, Bruna, and Giorgio Israel. 1990. *The Invisible Hand: Economic Equilibrium in the History of Science*. Cambridge, MA: MIT Press.

Israel, Giorgio, and Ana Millàn Gasca. 2009. *The World as a Mathematical Game: John von Neumann and Twenthieth Century Science*. Basel: Birkhauser.

Kass, Seymour. 1996. "Karl Menger." *Notices of the American Mathematical Society*, 43(5): 558-561.

Kjeldsen, Tinne H. 2001. "John von Neumann's Conception of the Minimax Theorem: A Journey through Different Mathematical Contexts." *Archives of the History of Exact Sciences*, 56(1): 39-68.

Kurz, Heinz D., and Neri Salvadori. 1993. "Von Neumann's Growth Model and the 'Classical' Tradition." *The European Journal of the History of Economic Thought*, 1(1): 129-159.

Leonard, Robert. 1995. "From Parlor Games to Social Science: Von Neumann, Morgenstern, and the Creation of Game Theory, 1928-1994." *Journal of Economic Literature*, 33(2): 730-761.

Leonard, Robert. 1998. "Ethics and the Excluded Middle: Karl Menger and Social Science in Interwar Vienna." *Isis*, 89(1): 1-26.

Leonard, Robert. 2010. *Von Neumann. Morgenstern and the Creation of Game Theory. From Chess to Social Science, 1900-1960*. Cambridge: Cambridge University Press.

Marchionatti, Roberto, and Enrico Gambino. 1997. "Pareto and Political Economy as a Science: Methodological Revolution and Analytical Advances in Economic Theory in the 1890's." *Journal of Political Economy*, 105(6): 1322-1348.

Marchionatti, Roberto. 1999. "The Methodological Foundation of Pure and Applied Economics in Pareto. An Anti-Walrasian Programme." *Revue européenne des sciences sociales*, 37(116): 277-294.

Marchionatti, Roberto, and Raffaella Fiorini. 2000. "L. Von Bortkievicz and the Origin of Neo-Ricardian Theory." *Revue européenne des sciences sociales*, 38(117): 173-191.

Marchionatti, Roberto, and Fiorenzo Mornati. 2003. "Pareto et l'économie mathématique au debut des années '90. Quelques reflections à propos des 'considerazioni sui principii fondamentali dell'economia politica pura'." In *Histoire et théorie des sciences sociales: mélanges en l'honneur de Giovanni Busino*, ed. Mohamed Cherkaoui, 351-376. Genève: Librairie Droz.

Marchionatti, Roberto. 2004. "The Classical Era of Mathematical Economics." In *Early Mathematical Economics, 1871-1915*, ed. Marchionatti Roberto, 1-63. London: Routledge.

Marchionatti, Roberto. 2006. "At the Origin of Post-War Mainstream of Economics: On Pareto's Influence on Economic Theory." *International Review of Economics*, 53(4): 538-559.

Menger, Karl. 1935. "Sull'indirizzo di idee e sulle tendenze principali del colloquio matematico di Vienna." *Annali della scuola normale superiore di pisa - classe di scienze*, 4(4): 327-339.

Menger, Karl. 1936. "Bemerkungen zu den Ertragsgesetzen." *Zeitschrift für Nationalökonomie*, 7(3): 388-397.

Menger, Karl. 1952. "The Formative Years of Abraham Wald and His Work in Geometry." *The Annals of Mathematical Statistics*, 23(1): 14-20.

Menger, Karl. 1974. *Morality, Decision, and Social Organization: Toward a Logic of Ethics*. Dordrecht: D. Reidel.

Menger, Karl. 1994. *Reminiscences of the Vienna Circle and the Mathematical Colloquium*. Dordrecht: Kluver Academic Publishers.

Mongin, Philippe. 2003. "L'axiomatisation et les théories économiques." *Revue économique*, 54(1): 99-138.

Morgenstern, Oskar. 1941. "Professor Hicks on Value and Capital." *Journal of Political Economy*, 49(3): 361-393.

Morgenstern, Oskar. 1951. "Abraham Wald, 1902-1950." *Econometrica*, 19(4): 361-367.

Morgenstern, Oskar. 1976. "The Collaboration between Oskar Morgenstern and John von Neumann on the Theory of Games." *Journal of Economic Literature*, 14(3): 805-816.

Mornati, Fiorenzo. 1999. "Le début des différends entre Pareto and Walras vus a travers leur correspondence et leur ouvrages: 1891-1893." *Revue européenne des sciences sociales*, 37(116): 261-276.

Mutoh, Isao. 2003. "Mathematical Economics in Vienna between the Wars." In *Advances in Mathematical Economics*, ed. Shigeo Kusuoka and Toru Maruyama, 167-195. Tokyo: Springer-Verlag.

Napoleoni, Claudio. 1965. *L'equilibrio economico generale. Studio introduttivo*. Torino: Boringhieri.

Neisser, Hans. 1932. "Lohnrohe und Beschäftigungsgrad in Marktgleichgewicht." *Weltwirtschaftliches Archiv*, 36: 415-455.

Neurath, Otto. 1973. *Empiricism and Sociology*. Dordrecht: D. Reidel.

Neurath, Otto. 2004. *Economic Writings. Selections 1904-1945*. Dordrecht: Kluwer Academic Publishers.

Pareto, Vilfredo. 1892a. "Considerazioni sui principi fondamentali dell'economia politica pura." *Giornale degli economisti*, 3(4): 389-420.

Pareto, Vilfredo. 1892b. "Considerazioni sui principi fondamentali dell'economia politica pura." *Giornale degli economisti*, 3(4): 485-512.

Pareto, Vilfredo. 1892c. "Considerazioni sui principi fondamentali dell'economia politica pura." *Giornale degli economisti*, 3(5): 119-157.

Pareto, Vilfredo. 1893a. "Considerazioni sui principi fondamentali dell'economia politica pura." *Giornale degli economisti*, 4(6): 1-37.

Pareto, Vilfredo. 1893b. "Considerazioni sui principi fondamentali dell'economia politica pura." *Giornale degli economisti*, 4(7): 279-321.

Pareto, Vilfredo. 1902. "Anwendungen der Mathematik auf Nationalökonomie." In *Enzyklopadie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, ed. Felix Klein and Wilhelm Meyer, 1094-1120. Leipzig: Deuticke.

Pareto, Vilfredo.1906. *Manuale di economia politica.* Milano: Società editrice libraria.

Pareto, Vilfredo. 1909. *Manuel d'économie politique.* Paris: Giard et brière.

Pareto, Vilfredo. 1917. "Discours du Jubilé." *Revue d'économie politique*, 31: 426-430.

Pareto, Vilfredo. 1984. *Lettere a Maffeo Pantaleoni (1890-1923)*. Genève: Librairie Droz.

Pomini, Mario. 2012. "From Steady-State to Dynamic Equilibrium: The Perspective in the Interwar Period." *History of Economic Ideas*, 20(2): 17-41.

Punzo, Lionello F. 1989. "Karl Menger's Mathematical Colloquium." In *John von Neumann and Modern Economics*, ed. Mohammed Dore, Sukhamoy Chakravarty, and Richard Goodwin, 129-165. Oxford: Clarendon Press.

Punzo, Lionello F. 1991. "The School of Mathematical Formalism and the Viennese Circle of Mathematical Economists." *Journal of the History of Economic Thought*, 13(1): 1-18.

Remak, Robert. 1929. "Kann die Volkswirtschftslehre eine exakte Wissenschaft werden?" *Jahrbücher für Nationalökonomie und Statistik*, 131: 703-735.

Remak, Robert. 1933. "Können superponierte Preissysteme praktisch berechnet werden?"*Jahrbücher für Nationalökonomie und Statistik*, 138: 839-842.

Samuelson, Paul A. 1941. "The Stability of Equilibrium: Comparative Statics and Dynamics." *Econometrica*, 9(2): 97-120.

Samuelson, Paul A. 1948. *Foundations of Economic Analysis*. Cambridge, MA: Harvard University Press.

Samuelson, Paul A. 1998. "How Foundations Came to Be." *Journal of Economic Literature*, 36(3): 1375-1386.

Schlesinger, Kurt. 1935. "Über die Produktionsgleichungen der Ökonomischen werlehre." In *Ergebnisse eines mathematischen Kolloquiums 6*, ed. Karl Menger, 10-11. Leipzig: Deuticke.

Schumpeter, Joseph A. 1954. *History of Economic Analysis*. London: Allen and Unwin.

Shackle, George L. S. 1967. *The Years of High Theory: Invention and Tradition in Economic Thought, 1926-1939*. Cambridge, UK: Cambridge University Press.

Slutsky, Eugen. 1915. "Sulla teoria del bilancio del consumatore." *Giornale degli economisti e rivista di statistica*, 51(1): 1-26.

Tinbergen, Jan. 1949. "Du système de Pareto aux modèles moderne." *Revue d'économie politique*, 59: 642-652.

Ulam Stanislaw. 1958. "John von Neumann, 1903-1957." *Bullettin of the American Mathematical Society*, 64(3): 1-49.

Von Hayek, Friedrich A. 1933. *Monetary Theory and the Trade Cycle.* London: Jonathan Cape.

Von Neumann, John. 1937. "Über ein ökonomisches Gleichungssystem und eine Verallgemeinerung des browerschen Fixpunktsatze." In *Ergebnisse eines mathematischen Kolloquiums 8*, ed. Karl Menger, 73-83. Leipzig: Deuticke.

Von Neumann, John.1945. "A Model of General Economic Equilibrium." *The Review of Economic Studies*, 13(1): 1-9.

Von Stackelberg, Heinrich. 1933. "Zwei kritische Bemerkungen zur preistheorie Gustav Cassel." *Zeitschrift der Nationalökonomie*, 4(4): 456-472.

Wald, Abraham. 1935. "Über die eindeutige positive Lösbarkeit der neuen Produktionsgleichungen." In *Ergebnisse eines mathematischen Kolloquiums 6*, ed. Karl Menger, 12-18. Leipzig: Deuticke.

Wald, Abraham. 1936a. "Über die Producktionsgleichungen der ökonomischen Wertlehre (Mitteilung II)." In *Ergebnisse eines mathematischen Kolloquiums 7*, ed. Karl Menger, 1-6. Leipzig: Deuticke.

Wald, Abraham. 1936b. "Über einige Gleichungssysteme der mathematischen Ökonomie." *Zeitschrift der Nationalökonomie*, 7(5): 637-670.

Walras, Léon.1900. *Eléments d'économie politique pure, ou théorie de la richesse sociale*. 4 édition. Lausanne: Rouge.

Weintraub, Eliot R. 1983. "On the Existence of a Competitive Equilibrium: 1930-1954." *Journal of Economic Literature*, 21(1): 1-39.

Weintraub, Eliot R. 1985.*General Equilibrium Analysis: Studies in Appraisal*. Cambridge, UK: Cambridge University Press.

Weintraub, Eliot R. 1998. "Controversy: Axiomatisches Mißverständnis." *Economic Journal*, 108(451): 1837-1847.

Weintraub, Eliot R. 2002. *How Economics Became a Mathematical Science*. Durham: Duke University Press.

Wittmann, Waldemar. 1967. "Die extremale Wirtschaft. Robert Remak – ein Vorläufer der Aktivitätsanalyse." *Jahrbücher für Nationalökonomie und Statistik*, 180(1): 397-409.

Young, Warren. 1991. "The Early Reactions to Value and Capital: Critics, Critiques and Correspondence in Comparative Perspective." *Review of Political Economy*, 3(3): 289-308.

Young, Warren. 2008. "Hicks in Reviews, 1932-1989: From 'The Theory of Wages' to 'A Market Theory of Money'." In *Markets*, *Money and Capital: Hicksian Economics for the Twenty-First Century*, ed. Roberto Scazzieri, Amartya K. Sen, and Stefano Zamagni, 109-126. Cambridge, UK: Cambridge University Press.

Zawadski, Władysław M. 1914. *Les mathématiques appliquées à l'économie politique*. Paris: Marcel Rivière & Cie.

Zeuthen, Frederik. 1932. "Das Prinzip der Knappheit, technische Kombination und ökonomische Qualität." *Zeitschrift der Nationalökonomie*, 4(1): 1-24.

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